Solutions 1
Decimals, Fractions, Percentage & Standard Form.

Decimals

1. $8.1 - 4.85 = \mathbf{3.25}$
2. $43 - 22.4 = \mathbf{20.6}$
3. $5.7 + 15.6 = \mathbf{21.3}$
4. $31.4 - 9.03 = \mathbf{22.37}$

Fractions

5. $6 + \frac{25}{30} + \frac{18}{30} = 6 + \frac{43}{30} = 7\frac{13}{30}$
6. $3 + \frac{6}{15} - \frac{10}{15} = 3 - \frac{4}{15} = 2\frac{11}{15}$
7. $\frac{11}{\cancel{4}^{1}} \times \frac{\cancel{4}^{1}}{3} = \frac{11}{3} = 3\frac{2}{3}$
8. $\frac{11}{2} \div \frac{11}{3} = \frac{\cancel{11}^{1}}{2} \times \frac{3}{\cancel{11}^{1}} = \frac{3}{2} = 1\frac{1}{2}$
9. $\frac{3}{8} \times \left(\frac{5}{3} - \frac{4}{7}\right) = \frac{3}{8} \times \left(\frac{35}{21} - \frac{12}{21}\right) = \frac{\cancel{3}^{1}}{8} \times \frac{23}{\cancel{21}^{7}} = \frac{23}{56}$
10. $\frac{3}{7} \times \left(\frac{11}{6} + \frac{3}{4}\right) = \frac{3}{7} \times \left(\frac{22}{12} + \frac{9}{12}\right)$
    $= \frac{\cancel{3}^{1}}{7} \times \frac{31}{\cancel{12}^{4}} = \frac{31}{28} = 1\frac{3}{28}$

Various

11. $23 + 36 \times \frac{3}{4} = 23 + \frac{\cancel{36}^{9}}{1} \times \frac{3}{\cancel{4}^{1}} = 23 + 27 = 50$
12. 10% is £85 $\times 3$ = £ 255
    1% = £8.50 $\times 2$ = £ 17
    32% is £ 272
13. $\frac{1}{8}$ of 544 is 68, so $\frac{3}{8}$ is $68 \times 3 = 204$

Using Percentages

1. $4500 \times 1.009^3 = 4622.59678\ldots$ **4620 (3 sf)**
2. $7000 \times 0.86^4 = 3829.0571\ldots$ **3830 (nst 10)**
3. House: £ 70 000 $\times 1.07^3$ = £ 85 753.01
   Contents: £ 45 000 $\times 0.91^3$ = £ 33 910.70
   Total value: = **£ 119 663.71**
4. Factory: £ 435 000 $\times 1.053^2$ = £ 482 331.92
   Plant & Mcy: £ 156 000 $\times 0.915^2$ = £ 130 607.10
   Total value: = **£ 612 939.02**
5. 66 $^2/_3$ % = $^2/_3$ So, $^2/_3$ off means you pay $^1/_3$
   They pay $^1/_3$ of £ 16.50 = £ 5.50
6. Percentage Increase = $\frac{1.4}{54.9} \times 100 = 2.55\%$
   Price in 2000 = 56.3×1.0255^4 = 62.3 p per litre

Reversing the change

7. Ex-VAT Price $\times 1.175$ = £695
   Ex-VAT Price = £695 $\div 1.175$ = **£ 591.49**
8. Stock $\times 0.4 = 50$ (60% sold = 40% left)
   Stock = $50 \div 0.4 = 125$
9. Original Price $\times 0.4$ = £ 4640
   Original Price = £ 4640 $\div 0.4$ = £ 11 600
10. Original Price $\times 0.875$ = £ 14 875
    Original Price = £ 14 875 $\div 0.875$ = £ 17 000

Standard Form

1. $8 \times 4.80 \times 10^8 = 3.84 \times 10^9$
2. $7.1 \times 10^7 \div 300 = 2.4 \times 10^5$
3. Time = Distance $\div$ Speed
   Time = $2.3 \times 10^8 \div 3.0 \times 10^5$
   Time = 766.67 sec = 13 minutes.
4. Distance = circumference = $2\pi r$
   Distance = $2\pi \times 0.6 \times 10^7$
   Speed = Distance $\div$ Time
   Time = $88 \times 24 = 2112$ hours
   Speed = $2\pi \times 0.6 \times 10^7 \div 2112$
   Speed = 17 849. 95… = 18 000 kph (2 sf)
5. $1.8 \times 10^3 \times 9.11 \times 10^{-31} = 1.6398 \times 10^{-27}$
   $= 1.6 \times 10^{-27}$ kg (2 sf)
6. $5 \times 10^6 \times 9.46 \times 10^{12}$ km
   $= 4.73 \times 10^{19}$ km
7. 1 year (not leap year) = $365 \times 24 \times 60 \times 60$
   = 31536000 seconds
   Profit = £ $3.2 \times 10^9 \div 31536000$ = £101.47133….
   = £ 101 per second.
8. No. of days = 26 (J) + 31(J) + 31(A) + 20 (S)
   = 108
   $2.925 \times 10^7 \div 108 = 270\ 833.333$
   = 270 833 visitors per day
9. $5.97 \times 10^{24} \div 2.2 \times 10^{30} \times 100$
   = 0.0002713…… %
   $= 2.71 \times 10^{-4}$ % (3 sf)

Solutions Algebra-1
Basic Algebraic operations

Evaluation

1. $30 - 3(-1)^2(-6) = \mathbf{48}$

Simplification

2. $12x - 8 - 20x - 5 \rightarrow \mathbf{-8x - 13}$

3. $6a^2 - 15ab - 2ab + 5b^2 \rightarrow \mathbf{6a^2 - 17ab + 5b^2}$

4. $2x^2 + 6x - x - 3 + x^2 - 8x + 16$
$\rightarrow \mathbf{3x^2 - 3x + 13}$

5. $(3y - 4)(3y - 4) \rightarrow \mathbf{9y^2 - 24y + 16}$

6.

	$3x^2$	$4x$	-1
$2x$	$6x^3$	$8x^x$	$-2x$
-3	$-9x^2$	$-12x$	3

$\rightarrow \mathbf{6x^3 - x^2 - 14x + 3}$

7. $3x(2x - 3)$

8. $(2a + 3b)(2a - 3b)$

9. a) $(3x + y)(3x - y)$

b) $\dfrac{6x+2y}{9x^2-y^2} \rightarrow \dfrac{2\cancel{(3x+y)}}{\cancel{(3x+y)}(3x-y)} \rightarrow \dfrac{2}{3x-y}$

10. a) $(a + 3b)(a - 3b)$

b) $\dfrac{a^2-9b^2}{2a+6b} \rightarrow \dfrac{\cancel{(a+3b)}(a-3b)}{2\cancel{(a+3b)}} \rightarrow \dfrac{a-3b}{2}$

11. a) $(x + 3)(x - 3)$

b) $\dfrac{4(5x+3)}{25x^2-9} \rightarrow \dfrac{4\cancel{(5x+3)}}{\cancel{(5x+3)}(5x-3)} \rightarrow \dfrac{4}{5x-3}$

12. $\dfrac{15x-20}{9x^2-16} \rightarrow \dfrac{5\cancel{(3x-4)}}{(3x+4)\cancel{(3x-4)}} \rightarrow \dfrac{5}{3x+43}$

13. a) $2x(x - 3)$

b) $\dfrac{2x^2-6x}{x^2-9} \rightarrow \dfrac{2x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} \rightarrow \dfrac{2x}{x+3}$

14. $3x^2 - 13x - 10 \rightarrow (3x+2)(x-5)$

15. $5 - 2 - 6x = 27 \quad -24 = 6x \quad x = -4$

16. $5 + 3a = a - 15 \quad 2a = -20 \quad a = -10$

17. $2a + 4b = -7$ ... (1)
$3a - 5b = 17$ ... (2)
multiply (1) x 5 and (2) x 4

then add to get a = 1½ , subst. to get b = -2½

18. $5a + 3b = 9$ ... (1)
$7a - 2b = 25$ ... (2)
multiply (1) x 2 and (2) x 3

then add to get a = 3 , subst. to get b = –2

Functions

1. $f(-2) = (-2)^2 - 2(-2) \rightarrow 4 + 4 \rightarrow 8$

2. $h(-2) = 15(-2) - 3(-2)^2 \rightarrow -30 - 12 \rightarrow -42$

3. $f(-3) = \dfrac{(-3)^3 + (-3)^2 + 2}{5(-3) - 1} \rightarrow \dfrac{-27 + 9 + 11}{-16} \rightarrow \dfrac{7}{16}$

4. a) $f(-3) = 9 - 6(-3) \rightarrow 9 + 18 \rightarrow 27$

b) $f(t) = 9 - 6t \quad 11 = 9 - 6t \quad 6t = -2 \quad t = -\dfrac{1}{3}$

5. a) $f(-2) = 3(-2)^2 - 7 \rightarrow 12 - 7 \rightarrow 5$

b) $f(a) = 3a^2 - 7 \quad 20 = 3a^2 - 7 \quad 3a^2 = 27$
$a^2 = 9 \quad a = 3 \ or \ a = -3$

6. $f\left(\dfrac{1}{2}\right) = \dfrac{4}{\left(\frac{1}{2}\right)^2} \rightarrow \dfrac{4}{\frac{1}{4}} \rightarrow 4 \div \dfrac{1}{4} \rightarrow 4 \times \dfrac{4}{1} \rightarrow 16$

7. a) $f(x) = 3^x \quad f(4) = 3^4 \rightarrow 81$

b) $\sqrt{27} = 3^x, \ \left(3^3\right)^{\frac{1}{2}} = 3^x \quad 3^{\frac{3}{2}} = 3^x \quad \therefore x = \frac{3}{2}$

8. $f(2) = \dfrac{3}{\sqrt{2}} \rightarrow \dfrac{3}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} \rightarrow \dfrac{3\sqrt{2}}{2}$

9. $f(12) = 3\sqrt{12} \rightarrow 3\sqrt{4 \times 3} \rightarrow 3\sqrt{4}\sqrt{3} \rightarrow 6\sqrt{3}$

Quadratic Equations

1. $x^2 - 7x = 0 \rightarrow x(x-7) = 0 \quad x = 0, \ x = 7$

2. $6y - y^2 = 0 \quad y(6 - y) = 0 \quad y = 0, \ y = 6$

3. $(2x+1)(x-5) = 0 \quad x = -\frac{1}{2}, \ x = 5$

4. $(2x-3)(x+5) = 0 \quad x = \frac{3}{2}, \ x = -5$

5. $(2x-3)(x+4) = 0 \quad x = \frac{3}{2}, \ x = -4$

6. $(2p-5)(p+2) = 0 \quad p = \frac{5}{2}, \ x = -2$

7. $5x + 3 = x^2 + 2x - 1 \rightarrow x^2 - 3x - 4 = 0$
$\rightarrow (x+1)(x-4) = 0 \rightarrow x = -1, \ x = 4$

Solutions Algebra-1
Basic Algebraic operations *(continued)*

Quadratic Equations *(continued)*

8. Use formula a = 2, b = -3, c = -4 : x = 2.4, -0.9

9. Use formula a = 1, b = 2, c = -6 : x = 1.6, -3.6

Inequalities

1. $8-x>3(2x+5) \rightarrow 8-x>6x+15 \rightarrow x<-1$

2. $3y<4-(y+2) \rightarrow 3y<2-y \rightarrow y<\frac{1}{2}$

3. $3-x+6<2x \rightarrow 9<3x \rightarrow x>9$

4. $6x-2<5-15x \rightarrow 21x<7 \rightarrow x<\frac{1}{3}$

5. $2+5x\geq 8x-16 \rightarrow 18\geq 3x \rightarrow x\leq 6$

6. $2-15x+10\geq 4-12x \rightarrow 12-15x\geq 4-12x$
$\rightarrow 8\geq 3x \rightarrow x\leq\frac{8}{3} \quad x\leq 2\frac{2}{3}$

x = 1 or 2 since x is a positive integer.

7. $3x+1\leq 5x+3 \quad -2\leq x \quad i.e. \quad x\geq -2$
$5x+3\leq x+23 \quad 4x\leq 20 \quad i.e. \quad x\leq 5$
Both are true, so x = {-2, -1, 0, 1, 2, 3, 4, 5}

Changing subject of formula

1. $5Y=6v-3w \rightarrow 6v=5Y+3w \rightarrow v=\frac{5Y+3w}{6}$

2. $3P=m-s \rightarrow m=3P+s$

3. $L-8=\frac{6}{Y} \rightarrow Y(L-8)=6 \quad Y=\frac{6}{L-8}$

4. $dt=k-m \rightarrow k=dt+m$

5. $Q-p^2=3T \rightarrow T=\frac{Q-p^2}{3}$

6. $M+3=R^2t \rightarrow \frac{M+3}{t}=R^2 \rightarrow \sqrt{\frac{M+3}{t}}=R$

7. $A^2=4b^2-c \rightarrow A^2+c=4b^2 \rightarrow b=\sqrt{\frac{A^2+c}{4}}$

8. a) $Q-t=2\sqrt{s} \rightarrow \frac{Q-t}{2}=\sqrt{s} \rightarrow \left(\frac{Q-t}{2}\right)^2=s$

b) $s=\left(\frac{3.5-2.2}{2}\right)^2 \rightarrow \left(\frac{1.3}{2}\right)^2 \rightarrow 0.65^2 \rightarrow 0.4225$

9. $F=f-\frac{fv}{s} \rightarrow \frac{fv}{s}=f-F \rightarrow fv=s(f-F) \rightarrow v=\frac{s(f-F)}{f}$

Algebraic Fractions

1. $\frac{1}{2x}-\frac{1}{3x} \rightarrow \frac{3}{6x}-\frac{2}{6x} \rightarrow \frac{1}{6x}$

2. $\frac{3}{x}+\frac{2-x}{x^2} \rightarrow \frac{3x}{x^2}-\frac{2-x}{x^2} \rightarrow \frac{4x-2}{x^2}$

3. $\rightarrow \frac{5(x-2)}{x(x-2)}-\frac{3x}{x(x-2)} \rightarrow \frac{2x-10}{x(x-2)} \rightarrow \frac{2(x-5)}{x(x-2)}$

Fraction Equations

1. $\frac{2x+1}{3}-\frac{x}{4}=2 \rightarrow 4(2x+1)-3x=24 \rightarrow x=4$

2. $\frac{x+4}{2}-\frac{2x+1}{3}=1 \rightarrow 3(x+4)-2(2x+1)=6 \rightarrow x=4$

3. $3x-\frac{5x+2}{2}=3 \rightarrow 6x-5x-2=6 \rightarrow x=8$

4. $\frac{x-3}{2}+\frac{2x-1}{3}=4 \rightarrow 3(x-3)+2(2x-1)=24 \rightarrow x=5$

5. $\frac{x-2}{3}-\frac{x}{2}=\frac{1}{4} \rightarrow 4(x-2)-6x=3 \rightarrow x=-5\frac{1}{2}$

6. $\frac{x}{2}-\frac{x+1}{3}=4 \rightarrow 3x-2(x+1)=24 \rightarrow x=26$

7. $\frac{m}{3}=\frac{1-m}{5} \rightarrow 5m=3-3m \rightarrow m=\frac{3}{8}$

Indices

1. 9 **2.** y^2 **3.** $a^{-4}+5a^3$ **4.** $2y^3$

5. y^3 **6.** b **7.** b + 1 **8.** $a^{\frac{3}{2}}+a^{-\frac{1}{2}}$

Surds

1. $5\sqrt{2}$ **2.** $2\sqrt{6}$ **3.** $\sqrt{3}$ **4.** $3\sqrt{2}$

5. $10\sqrt{2}$ **6.** $6\sqrt{2}$ **7.** $2\sqrt{3}-2$

8. $f(12)=3\sqrt{12} \rightarrow 3\sqrt{4\times 3} \rightarrow 3\sqrt{4}\sqrt{3} \rightarrow 6\sqrt{3}$

9. $\frac{3}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} \rightarrow \frac{3\sqrt{5}}{5}$

10. $\sqrt{\frac{3}{24}} \rightarrow \sqrt{\frac{1}{8}} \rightarrow \frac{1}{\sqrt{4\times 2}} \rightarrow \frac{1}{2\sqrt{2}} \rightarrow \frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{4}$

11. $f(2)=\frac{3}{\sqrt{2}} \rightarrow \frac{3\sqrt{2}}{\sqrt{2}\sqrt{2}} \rightarrow \frac{3\sqrt{2}}{2}$

12. $f\left(\frac{3}{2}\right)=4^{\frac{3}{2}} \rightarrow \left(\sqrt{4}\right)^3 \rightarrow 2^3 \rightarrow 8$

Solutions – Data Handling
Probability & Statistics

Simple Probability

1. a) P(7) = $\frac{5}{50} \rightarrow \frac{1}{10}$

   b) P(Blue 7) = $\frac{1}{50}$

2. 12 face cards, P(face card) = $\frac{12}{52} \rightarrow \frac{3}{13}$

3. a) P(green pencil) = $\frac{11}{20}$

   b) P(blue pencil) = $\frac{7}{19}$ (only 19 pencils left)

4. P(green AND red) = $\frac{25}{50} \times \frac{10}{50} \rightarrow \frac{1}{2} \times \frac{1}{5} \rightarrow \frac{1}{10}$

   (for independent events MULTIPLY probabilities).

5. a) P(W or D) = 0.2 + 0.5 = 0.7
   (add probabilities for mutually exclusive events)

   b) P(Lose) = 0.3

6. a) P(miss) = 0.2 (20%)

   b) P(3 hits in a row) = 0.8 x 0.8 x 0.8 = 0.512

   c) P(H,M,M) = 0.8 x 0.2 x 0.2 = 0.032

7. a) P(not defective) = 0.85 (85%)

   b) 5000 x 0.85 = 4250 should not be defective.

8. a) P(M,M,M) = ½ x ½ x ½ = $\frac{1}{8}$

   b) P(3 boys – first IS a boy) = 1 x ½ x ½ = ¼

Probability from relative Frequency

1. a) P(< 3 yrs old) = $\frac{310}{600} \rightarrow \frac{31}{60}$

   b) $4200 \times \frac{10}{600} \rightarrow \frac{42000}{600} \rightarrow \frac{420}{6} \rightarrow 70$

2. a) P(scenery) = $\frac{80}{500} \rightarrow \frac{8}{50} \rightarrow \frac{4}{25}$

   b) P(25 & facilities) = $\frac{23}{500}$

   c) P(not cost) = $\frac{215}{500} \rightarrow \frac{43}{100}$

3. a) P(still water) = $\frac{35}{110} \rightarrow \frac{7}{22}$

   b) P(< 20 & Fizzy) = $\frac{10}{110} \rightarrow \frac{1}{11}$

4. a) P(new car) = $\frac{40}{120} \rightarrow \frac{1}{3}$

   b) P(18-40 & used car) = $\frac{30}{120} \rightarrow \frac{1}{4}$

Statistical Diagrams

1. 25% contain fewer than 50 matches.
   (Lower quartile is 25%)

2. **A = 25 B = 29 C = 43**
   Range of men = 60 – 18 = 42
   Range of ladies = 21
   Low of Ladies = 22, so **C** = 21 + 22 = **43**

   Men's median = 44, Ladies median = 44 – 15 = 29

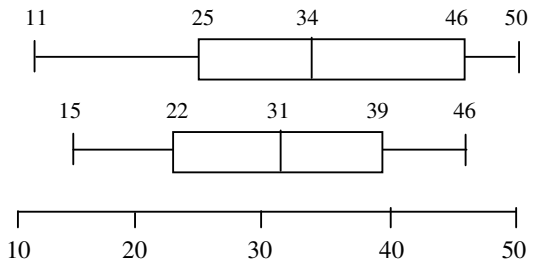
   S.I.R of men = (50 – 34) ÷ 2 = 16 ÷ 2 = 8
   S.I.R Ladies = ¾ of men's so it is 6
   So Ladies IQR = 6 x 2 = 12
   Subtract 12 from UQ to get 37 – 12 = 25.

3. Draw a boxplot for each one

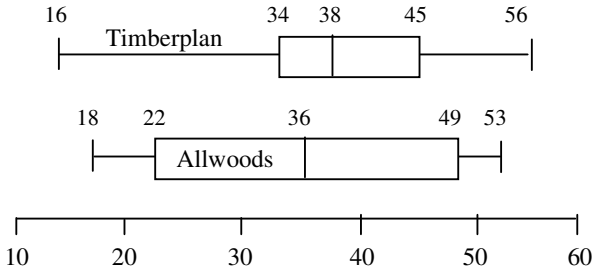
   Use Box plot (or back to back stem & leaf)

   1st Set:
   Lo = 11, Q_1 = 25, Q_2 = 34, Q_3 = 46, Hi = 50

   2nd Set:
   Lo = 15, Q_1 = 22, Q_2 = 31, Q_3 = 39, Hi = 46



4. a) Draw box plots



   b) The semi-interquartile range of timberplan is much lower than that of Allwoods, hence they are more consistent in their deliveries.

5. Draw a **Pie Chart**
   There are 30 pupils.
   Each one can be represented by 12°
   Walk = 156°, Bus = 108°, Car = 72°, Cycle = 24°
   (Check these **total 360°).**
   Then draw a **NEAT** pie chart, and **label** it.

6. a) There are 50 scores, so the median lies between the 25th and 26th scores.
i.e. between 73 and 75. **Median = 74**

b) (UQ – LQ) ÷ 2 LQ is 13th item
UQ is 38th item
So **S.I.R.** = (83 – 69) ÷ 2 = 14 ÷ 2 = **7**

c) Lo = 63, Hi = 98, Q_1=69, Q_2=74, Q_3=83

7. Put into order
6, 7, 9, 9, 12, 13, **16**, 18, 18, 20, 22, 24, 28
↑ LQ ↑ Median ↑ UQ

13 items: **Median** is 7th item = **16**
LQ= 9 UQ = 21
Transfer onto sketch.

6 9 16 21 28

Standard Deviation

1. Use formula $s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

Use 3 columns: x, $(x-\bar{x})$ $(x-\bar{x})^2$

$\sum x = 276$, mean = $276 \div 6 = 46$

$\sum(x-\bar{x})^2 = 84$, SD = $\sqrt{\frac{84}{5}} = 4.098...$

Mean = 46p Standard Deviation = 4.1p

Sugar prices more consistent compared to milk or milk prices more variable than sugar prices.

2. Use formula $s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

Use 3 columns: x, $(x-\bar{x})$ $(x-\bar{x})^2$

$\sum x = 102$, mean = $102 \div 8 = 12.75$

$\sum(x-\bar{x})^2 = 111.5$, SD = $\sqrt{\frac{111.5}{7}} = 3.991...$

A better formula to use is $s=\sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$
to avoid a lot of decimal calculations

$\sum x = 102$, $\sum x^2 = 1412$, $(\sum x)^2 = 10404$
This also gives SD = 3.991..

Mean = 12.75 hrs Standard Deviation = 3.99 hours

Alloa High School were more variable in the hours they spent in study time than Alloa Academy.

3. Use formula $s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

Use 3 columns: x, $(x-\bar{x})$ $(x-\bar{x})^2$

$\sum x = 750$, mean = $750 \div 5 = £150$

$\sum(x-\bar{x})^2 = 15200$, SD = $\sqrt{\frac{15200}{4}} = £61.64$

Mean = £150 Standard Deviation = £61.64

4. A better formula to use is $s=\sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$
to avoid a lot of decimal calculations

Mean = 84.33 pence Standard Deviation = 1.28 pence

The rural garages had a higher average price and the prices were more variable.

5. Use formula $s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

Use 3 columns: x, $(x-\bar{x})$ $(x-\bar{x})^2$

$\sum x = 36$, mean = $36 \div 6 = 6$

$\sum(x-\bar{x})^2 = 84$, SD = $\sqrt{\frac{80}{5}} = 4$

Mean = 6 Standard Deviation = 4

6. Use formula $s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

Use 3 columns: x, $(x-\bar{x})$ $(x-\bar{x})^2$

$\sum x = 78$, mean = $78 \div 6 = 13$

$\sum(x-\bar{x})^2 = 76$, SD = $\sqrt{\frac{76}{5}} = 3.898...$

Mean = 13 Standard Deviation = 3.9

Solutions

4. Area & Volume

1. a) $V = \pi r^2 h \rightarrow V = \pi \times 5^2 \times 14 = 1099.55...$
   V = 1100 (3 s.f.) Note radius is 5 cm.

   b) Cross section stays the same, height will change

   $600 = \pi \times 5^2 \times h \rightarrow h = \dfrac{600}{25\pi} \quad h = 7.6394...$
   depth of coffee = 7.6 centimetres (2 s.f.)

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2. Calculate volume of cylinder.
   $V = \pi r^2 h \rightarrow V = \pi \times 0.55^2 \times 1.85 = 1.75811..\text{m}^3$
   This volume will have been in the top tank.
   Cross section stays the same, height will change

   $1.7581 = 3 \times 9 \times h \rightarrow h = \dfrac{1.7581}{27} \quad h = 0.065114...$ m
   × 1000 to change to mm → 65.114… = 65 mm

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3. Volume of prism = Area cross section x length
   Area of rectangle = 0.6 x 0.25 m^2
   Area of semi-circle = ½ π $(0.3)^2$ (NB use radius)
   Area of cross section = 0.6 x 0.25 + ½ π $(0.3)^2$
   Area of cross section = 0.29137…
   Volume = 0.2913… x 4 = 1.1654… = 1.2 m^3 (2sf)

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4. Volume of prism = Area cross section x length
   Area of 2 rectangles = 2 x 4.5 x 2 = **18 m^2**
   Area of inner semi-circle = ½ π $(2)^2$
   Area of outer semi-circle = ½ π $(4)^2$
   Shaded area = ½ π $(4)^2$ - ½ π $(2)^2$ = **18.8495.. m^2**
   Area of cross section = 18 + 18.85 = 36.85 m^2
   Volume = 36.85 x 0.8 = 29.55 = 29.6 m^3 (1 dp)

*****Misprint – should ask for new diameter

5. a) $V = \pi r^2 h \rightarrow V = \pi \times 3.25^2 \times 15 = 497.746..$
   V = 497.75 (2 dp.) Note radius is 3.25 cm.

   b) Volume stays the same, height is reduced

   $497.75 = \pi \times r^2 \times 12 \rightarrow r^2 = \dfrac{497.75}{12\pi} \quad r^2 = 13.203...$
   radius = 3.663… hence diameter = 7.2672..
   new diameter = 7.3 cm (1 d.p.)

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6. Volume of prism = Area cross section x length
   Area of triangle = ½ a b sin C
   = ½ x 8 x 14 x sin 100° = 55.14923…
   Volume = 55.14923. x 5 = 275.746..
   Volume = = 276 cm^3 (2 sf)

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7. Volume of space = Vol. Cylinder – Vol cuboid
   Calculate volume of cylinder.
   $V = \pi r^2 h \rightarrow V = \pi \times 6^2 \times 20 = 720\pi \text{ cm}^3$
   Cross section of cuboid is a square

   Diagonal = 12 cm, Area = ½ diag x diag
   Area = ½ x 12 x 12 = 72 cm^2

   Or use Pythagoras
   Side of square = $\sqrt{6^2 + 6^2} = \sqrt{72}$ cm
   Area of square -= $\sqrt{72} \times \sqrt{72} = 72$ cm^2

   Volume of cuboid = 72 x 20 = 1440

   Hence vol of space = 720π - 1440
   = 720(π - 2)

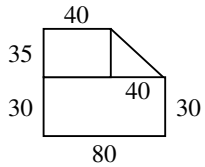
---

8. a) Dimensions of packet are < 1 litre
   Vol = 6 x 10 x 15 = 900 cm^3 < 1 litre

   b) Volume of cylinder
   $V = \pi r^2 h \rightarrow 900 = \pi \times 6^2 \times h \rightarrow h = \dfrac{900}{\pi \times 6^2}$
   h = 7.9577… 8.0 cm (1 d.p.)

---

9. Cross section is rectangle + trapezium
   or 2 rectangles and a triangle



   NB answer is in cubic metres.
   So work in metres
   Area of cross section :
   Area Trapezium
   = ½ (0.8 +0.4)x 0.35 = 0.21 m^2
   Area rectangle = 0.3 x 0.8 = 0.24 m^2
   Cross section area = 0.21 + 0.24 = 0.45 m^2
   Or
   Area of rectangles = 0.3 x 0.8 + 0.35 x 0.4 = 0.38
   Area of triangle = ½ x 0.4 x 0.35 = 0.07
   Cross section area = 0.38 + 0.07 = 0.45 m^2

   Volume = 0.45 x 1 = 0.45 m^3

---

10. a) x + x + 1.5 = 3, so 2x = 1.5, x = 0.75 m
    b) Cross section area: 2 triangles + 2 rectangles

    Area Rect: 3 x 0.6 + 1.5 x 0.8 = 3 m^2
    Area Triangles = 2 x ½ x 0.75 x 0.8 = 0.6 m^2

    Cross section area = 3 + 0.6 = 3.6 m^2

    Volume = 3.6 x 2 = 7.2 cubic metres.

Solutions
4. Area & Volume

11. Find area of cross section
Rectangle + semi circle
Area Rectangle = 7 x 5 = 35 m^2
Area semi-circle = ½ π 3.5^2 = 19.24 m^2

Area of cross section = 35 + 19.24 = 54.24 m^2

Volume of barn = 12 x 54.24 = 650.88… m^3
= 650 m^3 (2 sig. figs.)

---

12. Volume of prism = Area cross section x length
Area of triangle = ½ base x height (base = 6.5 – 0.5)
= ½ x 6 x 2 = 6 m^2
Area of rectangle = 2 x 0.5 = 1 m^2
Area of cross-section = 6 + 1 = 7 m^2
Volume = 7 x 3 = 21 m^3

---

13. **Area of existing cross section:**
Area of Rectangle = 14 x 5 = 70 m^2
Area of triangles = 2 x ½ x 10 x 5 = 50 m^2
Area of cross section = 120 m^2

Area of new cross section:
Area of Rectangle = 22 x 5 = 110 m^2
Area of triangles = 2 x ½ x 10 x 5 = 50 m^2
Area of cross section = 160 m^2

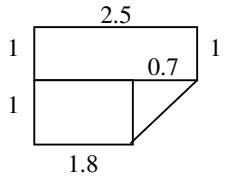
Area of cross section to be removed:
= 160 – 120 = 140 m^2

Volume to be removed = A x *l*
= 140 x 10 000 = 1 400 000 m^3 (change km to m)

Cost at £4 per m^3 = 4 x 1 400 000 = £5,600,000

---

14. Area of cross section:



Area of Rectangles:
= 1 x 2.5 + 1 x 1.8
= 4.3 m^2

Area of Triangle
= ½ x 0.7 x 1 = 0.35 m^2

Cross section area = 4.3 + 0.35 = 4.65 m^2

Volume = A x *l*

= 4.65 x 2 = 9.3 m^3

Solutions
5 Similar Shapes

1. Linear scale factor = $\frac{9}{6} \rightarrow \frac{3}{2}$

   Volume: = $30 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = 101.25\ mls$

---

2. Linear scale factor = $\frac{40}{50} \rightarrow \frac{4}{5}$

   Area: = $3.27 \times \frac{4}{5} \times \frac{4}{5} = 2.0928\ m^2$ = 2.09 m^2

---

3. Linear scale factor = $\frac{27}{18} \rightarrow \frac{3}{2}$

   Cost: = $80 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = £2.70$

---

4. Linear scale factor = $\frac{30}{20} \rightarrow \frac{3}{2}$

   Volume: = $0.8 \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = 2.7\ litres$

---

5. Linear scale factor = $\frac{24}{30} \rightarrow \frac{4}{5}$

   Volume: = $1.2 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = 0.6144\ litres$

   Volume = 0.61 litres (2 sig figs)

---

6. Linear scale factor = $\frac{200}{160} \rightarrow \frac{5}{4}$

   Cost: = $£1.12 \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = £2.1875$

   Cost = £ 2.19

Similar Triangles

1. Due to parallel line, Triangles are similar

   $\frac{BP}{6} = \frac{1}{1.5} \rightarrow BP = \frac{6}{1.5} = 4$

   Hence AP = 6 – 4 = 2 metres

   In Figure 2 the triangles are similar:

   Let B be h metres above the ground

   $\frac{h}{1} = \frac{6}{AP}$ but $AP = 2$ $\frac{h}{1} = \frac{6}{2}$ $h = 3\ metres$

---

2. a) Use converse of Pythagoras in Δ ABX
   $AB^2 = 300^2 = 90000$
   $AX^2 + BX^2 = 180^2 + 240^2 = 90000$
   Since $AB^2 = AX^2 + BX^2$ then ∠ AXB is 90°
   So roads AX and BX are at right angles to one another

   b) Shortest route is AX → XC → CD
   Triangles ABX and XCD are similar
   ∠A = ∠D, ∠B = ∠C (alternate angles)
   So,
   $\frac{XC}{240} = \frac{750}{300} \rightarrow XC = \frac{750 \times 240}{300} = 600$
   Shortest distance = 180 + 600 + 750 m
   = 1530 metres = 1.53 km

---

3. AC = 24 cm (diameter) and ∠ACD = 58°
   Using SOH-CAH-TOA,
   $\sin 58 = \frac{AD}{24} \rightarrow AD = 24 \sin 58 = 20.35\,cm$

   Δ AEO and ΔADC are similar (parallel line)

   $\frac{AE}{20.35} = \frac{12}{24} \rightarrow AE = \frac{1}{2} \times 20.35 = 10.175$

   Hence ED = 20.35 – 10.175 = 10.175 cm

---

4. ∠B = 80° (angle sum triangle ABC)
   ∠E = 65° (angle sum triangle DEF)
   Triangles are equiangular, hence similar.

   $\frac{DE}{4.6} = \frac{10.5}{4.2} \rightarrow DE = \frac{10.5 \times 4.6}{4.2} = 11.5$
   Hence DE = 11.5 centimetres

---

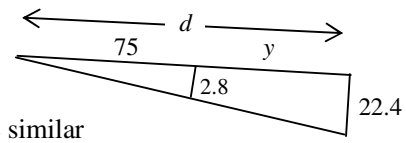
5. $\frac{BE}{6} = \frac{10}{12} \rightarrow BE = \frac{10 \times 6}{12} = 5$ cms

---

6. $\frac{CD}{8.4} = \frac{3}{4.5} \rightarrow CD = \frac{3 \times 8.4}{4.5} = 5.6$ m

---

7.



   The triangles are similar (parallel line).

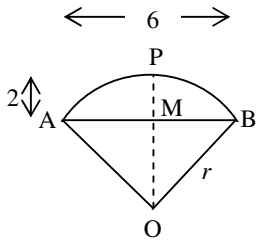
   $\frac{d}{75} = \frac{22.4}{2.8} \rightarrow d = \frac{22.4 \times 75}{2.8} = 600\ cms$

   Hence y = 600 – 75 = 525 centimetres

   So, distance from top of 10p coin to top of person's head is 525 centimetres.

Solutions

6 Pythagoras



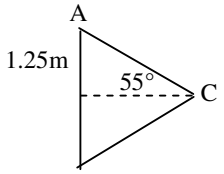
1. Let mid-point of AB be M and radius OB be r

   MB = 3m (½ of width)
   OP = r (radius)
   OM = r - 2

   By Pythagoras: $r^2 = (r-2)^2 + 3^2$
   So, $r^2 = r^2 - 4r + 4 + 9$
   4r = 13, r = 3.25 metres.

---

2. a) Draw triangle as shown



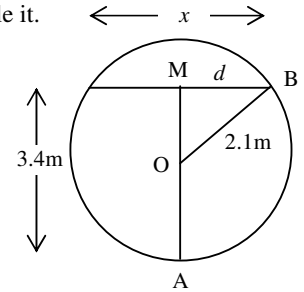
   $\sin 55° = \dfrac{1.25}{AC}$

   $AC = \dfrac{1.25}{\sin 55°} = 1.5259...$ hence AC = 1.53m (3 sf)

   b) Find perimeter of table
   radius of arc = 1.53m (diameter = 3.06m)
   length of curved end = $\dfrac{110}{360} \times \pi \times 3.06 = 2.94$ m
   length of straight section = 5m
   Perimeter = 2.94 + 2.94 + 5 + 5 = 15.88m
   = 1588 cm
   No. of people = 1588 ÷ 75 = 21.17.. So **21 people**

---

3. a) Find MB and then double it.



   OA = 2.1m (radius)

   OM = 3.4 – 2.1 m
   = 1.3 m

   By Pythagoras

   $2.1^2 = 1.3^2 + d^2$

   Hence $2.1^2 - 1.3^2 = d^2$ so d = 1.649..
   Hence x = d × 2 = 3.298… = 3.3 m (2 s.f.)

   b) By symmetry, the oil width will be the same, when it is below the centre by a distance OM.

   i.e. 2.1 – 1.3 = 0.8 metres.

---

4. a) CQ = radius = 10 cm
   By Pythagoras: $10^2 = x^2 + 8^2$ hence $10^2 - 8^2 = x^2$
   and so, x = 6 cm

   b) Height of figure = Ht. of Triangle + x + 10
   Let height of triangle be h, using symmetry:
   $h^2 + 8^2 = 17^2$
   So h = 15 cm
   Height of figure = 15 + 6 + 10 cm = **31 cms**

---

5. a) Use converse of Pythagroas
   AB^2 = 90 000
   $AX^2 + BX^2 = 180^2 + 240^2$ = 90 000
   Since $AB^2 = AX^2 + BX^2$ then AXB is 90°
   (converse of Pythagoras)
   So AX and BX are at right angles to each other.

   b) Shortest route is AX, XC, CD
   Need to find XC
   Triangles ABX and CXD are similar
   (using alternate angles on the parallel lines AB, CD)

   $\dfrac{XC}{240} = \dfrac{750}{300}$ hence XC = 600m
   Length of shortest route
   = 180 + 600 + 750 = 1530 metres.

---

6. Use converse of Pythagoras
   $d^2 = 37.3^2$ = 1391.29
   $22.5^2 + 30^2$ = 506.25 + 900 = 1406.25
   Since $d^2 \neq 22.5^2 + 30^2$ then AXB is NOT 90°
   (converse of Pythagoras)
   So No, the frame is NOT rectangular.

---

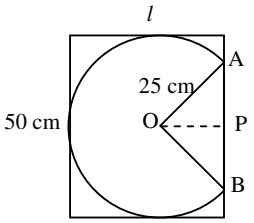
7. Find length of BD: OD = radius = 60 cm
   BOD is a right angled triangle
   Use Pythagoras: $BD^2 = 60^2 + 60^2$
   BD = 84.852…. cm = 84.9 cm (1 dp)

   Perimeter of circular part of table = $\dfrac{270}{360} \times \pi \times 120$
   = 282.7433.. cm = 282.7 cm (1 dp)

   Perimeter of table = 84.9 + 282.7 = 367.6 cm

---

8. Area of sector = $\dfrac{280}{360} \times \pi \times 50$ = 122.17.. = 122.2 cm^2



   To find l, we need to find OP and then add to the radius of 25 cm
   Angle AOB = 360 – 280 = 80°
   Angle AOP = 40°
   Using SOH-CAH-TOA

   $\cos 40 = \dfrac{OP}{25}$ hence OP = 19.151.. = 19.2 cm
   so min. length l reqd. = 19.2 + 25 = **44.2 cms.**

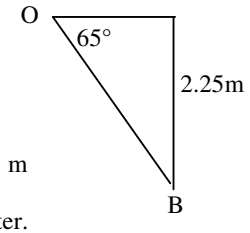
---

9. If they are to meet then this will be the angle in a semi-circle which should be 90°
   Use converse of Pythagoras
   4.1^2 = 16.81
   $2.6^2 + 3.1^2$ = 16.37
   Since $4.1^2 \neq 2.6^2 + 3.1^2$ then angle at top of bridge is NOT 90° (converse of Pythagoras).
   So beams will NOT fit this archway.

Solutions

6 Pythagoras *(continued)*

10. a) Use SOH-CAH-TOA



$\sin 65 = \dfrac{2.25}{OB}$

hence OB = 2.482.. = 2.48 m

b) Length of border = perimeter.

radius of curved edge = 2 x 2.48 = 4.96 m

Curved length = $\dfrac{130}{360} \times \pi \times 4.96$ = 5.626.. m

Length of border = 8.3 + 8.3 + 4.5 + 5.6 = 26.7 m

---

11. Height of tunnel
= distance O to the floor + O to top (the radius).
Let distance O to floor be d

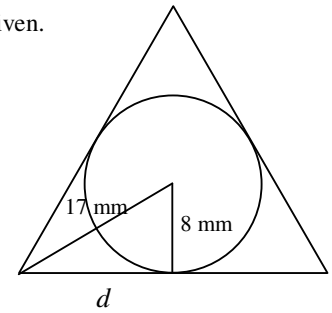
Use Pythagoras and symmetry:

$2.5^2 = d^2 + 1.2^2$ hence $d = 2.193\ldots$

Hence height of tunnel = 2.19 + 2.5 = 4.7 m (2 sf)

---

6. First draw the diagram
Mark on information given.



Find d.

$17^2 = d^2 + 8^2$

d = 15 cm

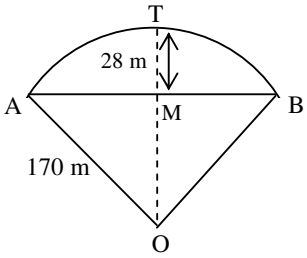
Hence each side
of triangle = 30 cm

Since equilateral, perimeter = 3 x 30 = 90 cms.

---

10. Converse of Pythagoras
14.5^2 = 210.25
$11.6^2 + 8.7^2 = 210.25$
Since $14.5^2 = 11.6^2 + 8.7^2$ then
angle is a perfect right angle (converse of Pythagoras),
so yes, it will be acceptable.

11.



OT = 170 (radius)
OM = 170 – 28 = 142

Use Pythagoras

$170^2 = AM^2 + 142^2$

AM = 93.466….

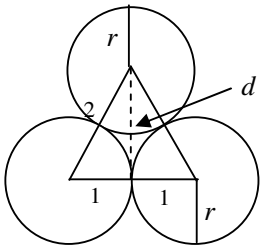
Hence AB
= 2 x 93.466..
= 187 m (3 sf)

---

12. Converse of Pythagoras
AC = 6 – (2 + 2.5) = 1.5 m
$AB^2 = 2.5^2 = 6.25$
$AC^2 + CB^2 = 2^2 + 1.5^2 = 6.25$
Since $AB^2 = AC^2 + CB^2$ then angle ACB = 90°
(converse of Pythagoras).

---

13. Height of stacked pipes =
radius + radius
+ distance between centres
of two layers



Triangle is equilateral,
by symmetry
and sides are all 2 m

By Pythagoras: $2^2 = 1^2 + d^2$ so $d = 1.732\ldots$ m

Hence height of pipe stack = 1 + 1 + 1.73 **= 3.73 m**

---

12. a) If AB = 2 then BC = 2 (it is a square)
By Pythagoras: $AC^2 = 2^2 + 2^2$ so AC = $\sqrt{8} \rightarrow 2\sqrt{2}$

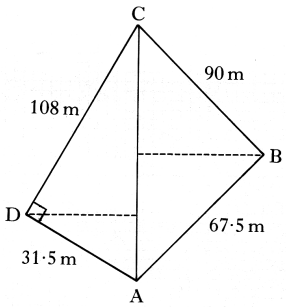
b) In any square of side a.

Diagonal = $\sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

Ratio of side to diagonal is: a : a√2
which is 1 : √2

---

13. Since angle ADC = 90°
By Pythagoras
$AC^2 = 108^2 + 31.5^2$
= 12656.25
$AB^2 + BC^2 = 90^2 + 67.5^2$
= 12656.25
Hence angle ABC = 90°
(converse of Pythagoras)



---

14. a) If d = 2, then using Pythagoras

$R^2 = 6^2 + (R-2)^2 \qquad R^2 = 36 + R^2 - 4R + 4$

Hence 4R = 40 and so R = 10

b) Volume = Volume of Cap + cylinder

Cap: $V = \dfrac{1}{3}\pi(2)^2(3\times 10 - 2) = \dfrac{112}{3}\pi$

Cylinder: $V = \pi \times \dfrac{5}{2} \times \dfrac{5}{2} \times \dfrac{8}{1} = 50\pi$

Total volume = $\dfrac{150}{3}\pi + \dfrac{112}{3}\pi = \dfrac{262\pi}{3}$

Solutions
7 The Circle

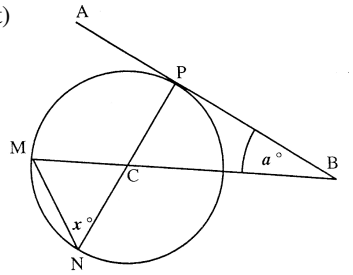
1. Area of sector = $\frac{40}{360}\times\pi\times15^2$ = 78.5 cm^2 (3 sf)

---

2. If they are to meet then this will be the angle in a semi-circle which should be 90°
Use converse of Pythagoras
$4.1^2 = 16.81$
$2.6^2 + 3.1^2 = 16.37$
Since $4.1^2 \neq 2.6^2 + 3.1^2$ then angle at top of bridge is NOT 90° (converse of Pythagoras).
So beams will NOT fit this archway.

---

3. ∠ CPB = 90° (tangent)



∠ PCB = 180 - a
(angle sum triangle)
∠ MCN = ∠ PCB
= 180 - a
(vertically opposite)
∠CMN = x°
(isosceles triangle)

So, $x + x + 180 - a = 180$ (angle sum of triangle)
Re-arrange to give: $2x = a$ So, $x = ½ a$

---

4. Area of sector = $\frac{50}{360}\times\pi\times1.2^2$ = 0.63 m^2 (2 sf)

---

5. Let angle of sector = θ So, $200 = \frac{\theta}{360}\times\pi\times15^2$

Re-arrange to get $\theta = \frac{200\times360}{\pi\times15^2} = \frac{320}{\pi}$

Length of arc: $\frac{\theta}{360}\times\pi\times30 = \frac{320}{\pi}\times\frac{\pi\times30}{360}$ = 26.7 m

Alternatively, $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{area of sector}}{\text{area of circle}}$

So, $\frac{\text{arc length}}{\pi\times30} = \frac{200}{\pi\times15\times15}$, arc length $= \frac{200\times\pi\times30}{\pi\times15\times15}$

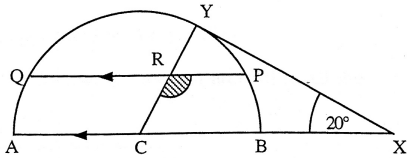
arc length = 26.7 m

---

6. Area of sector = $\frac{105}{360}\times\pi\times40^2$ = 1466 cm^2 (4 sf)
Area of screen (trapezium) = ½ (120 + 80) × 60
= 6000 cm^2

Area not cleaned = 6000 – 1466 = 4534 cm^2

7.



∠CYX = 90° (tangent), so ∠ YCX = 70° (angle sum Δ)
∠YRP = 70° (corresponding angle)
So, shaded angle
∠CRP = 110° (supplementary angle – adds up to 180°)

---

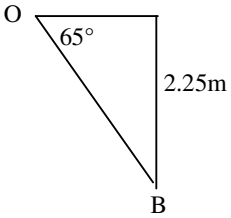
8. Find length of BD: OD = radius = 60 cm
BOD is a right angled triangle
Use Pythagoras: $BD^2 = 60^2 + 60^2$
BD = 84.852…. cm = 84.9 cm (1 dp)

Perimeter of circular part of table = $\frac{270}{360}\times\pi\times120$
= 282.7433.. cm = 282.7 cm (1 dp)

Perimeter of table = 84.9 + 282.7 = 367.6 cm

---

9. a) Use SOH-CAH-TOA



$\sin 65 = \frac{2.25}{OB}$

hence OB = 2.482.. = 2.48 m

b) Length of border = perimeter.

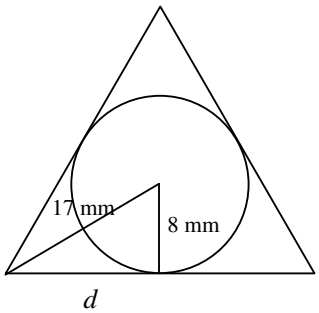
radius of curved edge = 2 x 2.48 = 4.96 m
Curved length = $\frac{130}{360}\times\pi\times4.96$ = 5.626.. m
Length of border = 8.3 + 8.3 + 4.5 + 5.6 = 26.7 m

---

10. Arc length = $\frac{160}{360}\times\pi\times60$ = 83.8 cm (3 sf)

---

11. First draw the diagram
Mark on information given.



Find d.

$17^2 = d^2 + 8^2$

d = 15 cm

Hence each side of triangle = 30 cm

Since equilateral, perimeter = 3 x 30 = 90 cms.

---

12. Let angle of arc = θ
Hence, $\frac{\theta}{360} = \frac{7}{\pi\times12}$, so θ = 66.8°

Angle through which the rod swings is 67°

Solutions
7 The Circle *(continued)*

13. Area of sector = $\frac{240}{360} \times \pi \times 3^2$ = 18.8 m^2 (3 sf)

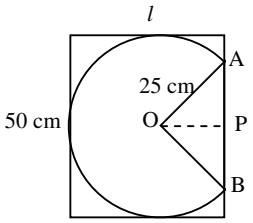
---

14. Length of waist = $\frac{140}{360} \times \pi \times 56$ = 68.4 cm (3 sf)

NB. the other dimension is not relevant to the question.

---

15. Area of sector = $\frac{280}{360} \times \pi \times 50$ = 122.17.. = 122.2 cm^2



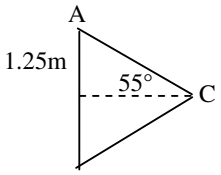
To find *l*, we need to find OP and then add to the radius of 25 cm

Angle AOB = 360 – 280 = 80°
Angle AOP = 40°
Using SOH-CAH-TOA

$\cos 40 = \frac{OP}{25}$ hence OP = 19.151.. = 19.2 cm

---

16. a) Draw triangle as shown



$\sin 55° = \frac{1.25}{AC}$

$AC = \frac{1.25}{\sin 55°} = 1.5259...$ hence AC = 1.53m (3 sf)

b) Find perimeter of table
radius of arc = 1.53m (diameter = 3.06m)
length of curved end = $\frac{110}{360} \times \pi \times 3.06 = 2.94\,\text{m}$
length of straight section = 5m
Perimeter = 2.94 + 2.94 + 5 + 5 = 15.88m
= 1588 cm
No. of people = 1588 ÷ 75 = 21.17.. So **21 people**

Solutions

8 Trigonometry – SOH-CAH-TOA

1. Find SV and then SW

$$\sin 34 = \frac{SV}{13.1} \quad \text{hence SV = 7.33 cms}$$

$$\cos 25 = \frac{SW}{SV} = \frac{SW}{7.33} \quad \text{so SW = 6.6 cms (2 sf)}$$

---

2. Let Length of ladder = l

$$\sin 60 = \frac{14}{l} \quad l = 16.17 \text{ m}$$

Look at second triangle, cat is 15 m up the tree.

Let angle of ladder be θ

$$\sin\theta = \frac{15}{16.17} \qquad \sin\theta = 0.9276 \qquad \theta = 68°$$

---

3. Let angle of ramp be θ

$$\tan\theta = \frac{0.5}{1.9} \rightarrow \theta = \tan^{-1}\left(\frac{0.5}{1.9}\right) \quad \theta = 14.7°$$

Yes, the ramp satisfies local building regulations.

---

4. a) B to C: $\tan 70 = \frac{13.5}{BC}$ BC = 4.9 m

b) A to B is AC – BC

$$\tan 40 = \frac{13.5}{AC} \quad \text{AC = 16.1 m}$$

Hence AB is: 16.1 – 4.9 = 11.2 metres.

---

5. a) Let diagonal of courtyard = d metres

$$\tan 8° = \frac{4.6}{d} \rightarrow d = \frac{4.6}{\tan 8°} = 32.7 \text{ metres}$$

b) Let length of side of courtyard = l metres.

Then by Pythagoras:

$$l^2 + l^2 = 32.7^2 \rightarrow 2l^2 = 1069.29$$

$$l = \sqrt{534.65} = 23.12... \text{ metres}$$

This is approx 23 metres.

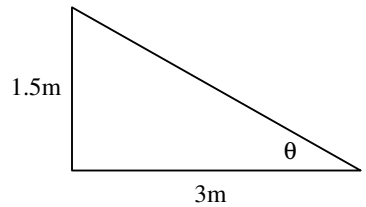
---

6. See Pythagoras Section 6
For the solution – misplaced in wrong section.

7.

$$\tan\theta = \frac{1.5}{3}$$

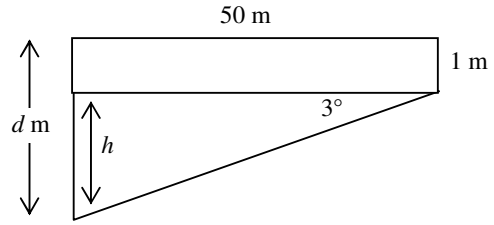
$$\theta = \tan^{-1}\frac{1.5}{3}$$



θ = 26.565.. °

Yes Planning permission should be granted, since angle is between 23° and 27°

8.



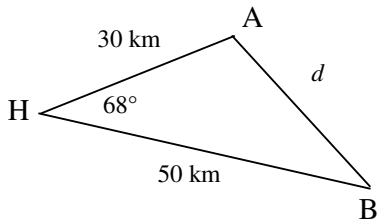
$$\tan 3° = \frac{h}{50} \quad \text{hence h = 50 tan 3° = 2.62 metres}$$

Hence d = 1 + 2.62 = 3.62

Hence depth of pool = 3.6 metres (2 sf)

Solutions
9 Trigonometry – Sine, Cosine Rule

1. Draw a diagram, and mark in given bearings which show that ∠AHB = 68



Look at diagram - SAS - Cosine Rule

$$d^2 = 30^2 + 50^2 - 2\times30\times50\times\cos 68°$$

d^2 = 3400 – 1123.819… = 2276.181…

d = 47.70933…..

yachts are 47.7 km apart when they stopped.

---

2. Area of triangle =1/2 a b sin C

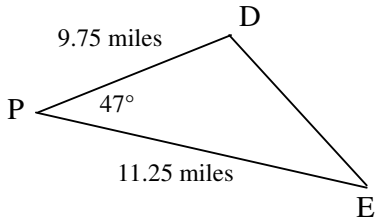
Transpose letters.

38 = ½ × 9 × 14 × sin B 38 = 63 sin B

Re-arrange: $\sin B = \frac{38}{63}$ B = $\sin^{-1}$ (38 ÷ 63)

Hence B = 37.096…. B = 37°

3.



PD = 13 × 0.75 = 9.75 miles

PE = 15 × 0.75 = 11.25 miles

∠DPE = 104°- 57° = 47°

Use cosine rule

$$DE^2 = 9.75^2 + 11.25^2 - 2\times9.75\times11.25\times\cos 47°$$

DE = 8.485… Boat D will have to travel 8 miles

---

4. Area = ½ a b sin C

So, 36 = ½ × 6 × 16 × sin R

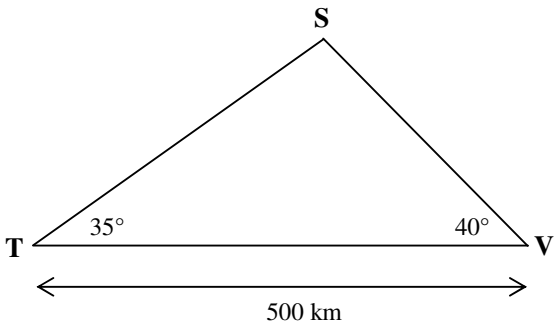
Hence $\sin R = \frac{36}{48} = \frac{3}{4}$

---

5. Use cosine Rule

$$\cos A = \frac{4^2 + 5^2 - 6^2}{2\times4\times5} = \frac{5}{40} = \frac{1}{8}$$

6.

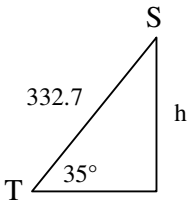


ASA - use Sine Rule to find either side ST or SV
The use SOH-CAH-TOA to find perpendicular height.

First find angle at S = 180° – (35° + 40°) S is 105°

$$\frac{ST}{\sin 40} = \frac{500}{\sin 105}$$

$$ST = \frac{500 \sin 40}{\sin 105} \Rightarrow ST = 332.731...$$



$$\sin 35 = \frac{h}{332.7}$$

$$h = 332.7\times\sin 35 = 190.828...$$

height of satellite = 190 km

---

7. Basically same as previous question

∠PRQ=95° Find RQ using sine rule

$$\frac{RQ}{\sin 50} = \frac{80}{\sin 95}$$ RQ = 61.5 metres

Now use SOH-CAH-TOA to find distance

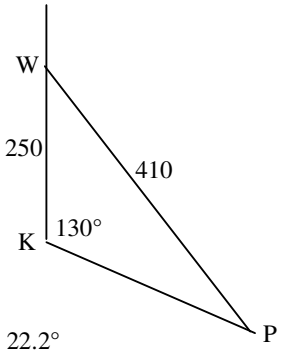
Let distance between river and path be *d* metres.

$$\sin 35 = \frac{d}{61.5}$$ hence, d = 35.3 metres

---

8. Draw diagram

Use sine rule to calculate angle at P.



$$\frac{\sin P}{250} = \frac{\sin 130}{410}$$

Hence sin P = 0.4671..
So, ∠ P = 27.8°

∠ KWP = 180 – (27.8 + 130) = 22.2°

Hence external angle = 157.8°

Bearing of Possum from Wallaby = 157.8°

Solutions
9 Trigonometry – Sine, Cosine Rule *(continued)*

9. Draw a larger diagram of required triangles

a) Use cosine rule: (let obtuse angle = θ)

$$\cos\theta = \frac{14^2 + 12^2 - 21^2}{2\times14\times12} = -\frac{101}{336}$$

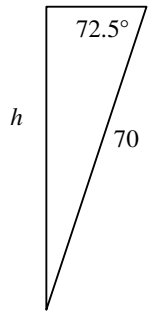
Hence acute θ = 72.5°,

so obtuse angle = 180 – 72.5 = 107.5°

b) Use SOH-CAH-TOA

Length of leg = 70 cms

Let height of table = h cms.



$$\sin 72.5 = \frac{h}{70}$$

hence h = 66.760… cms

height of table = 66.8 cms.

---

10. This is exactly the same as Qu. 6

Height of B = 112.3 metres

---

11. Use cosine Rule:

$$PR^2 = 101^2 + 98^2 - 2\times101\times98\times\cos 57°$$

PR = 94.99… = 95 cms.

---

12. $14 = \frac{1}{2}\times6\times7\times\sin A$

$\sin A = \frac{14}{21} = \frac{2}{3}$ acute A = 41.8°

Using ASTC, the sine is positive in 2nd quadrant.

Hence there is an angle 180 – 41.8 = 138.2°

Angles are: 42° and 138°

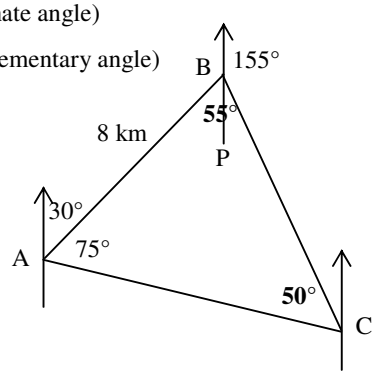
---

13. ∠ ABP = 30° (alternate angle)

∠ PBC = 35° (supplementary angle)

Hence,
∠ ABC = 55°

Also
∠ ACB = 50°
(angle sum triangle)



Use Sine Rule

$\frac{BC}{\sin 75} = \frac{8}{\sin 50}$ hence BC = 10.087…

Distance between B and C = 10.1 km (3 sf)

---

14. Area of triangle = ½ a b sin C

3rd angle of triangle = 65°

Area = ½ × 7 × 11 × sin 65° = 34.9 cm^2

---

15. a) ∠ RB South = 120° (alternate angles)

∠ YB South = 40° (since North B South = 180°)

Hence, ∠ RBY = 120° - 40° = 80°

b) Use cosine rule for RY

$$RY^2 = 350^2 + 170^2 - 2\times350\times170\times\cos 80°$$

RY = 361.6 km.

The people on the boat will be rescued first.

---

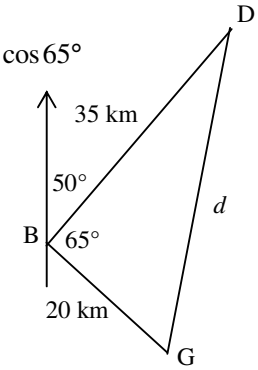
16. ∠ GBD = 125° - 50° = 65°

Use cosine rule to calculate d

$$d^2 = 35^2 + 20^2 - 2\times35\times20\times\cos 65°$$

Hence d = 32.145…

Distance between Delta and Gamma is 32 km.



---

17. Find 3rd angle in triangle = 114°

Let longer sloping edge (opp. 42°) be d metres

Use sine rule:

$\frac{d}{\sin 42} = \frac{12.8}{\sin 114}$ d = 9.375…

Length of longer sloping edge = 9.4 metres

---

18. Use cosine Rule

$$BC^2 = 420^2 + 500^2 - 2\times420\times500\times\cos 52°$$

BC = 409.66….

Hence BC = 410 metres (3 sf)

---

19. This is exactly the same as Qu. 6

Height of aeroplane = 16.6 metres

---

20. Area ΔPQS = ½ × 62 × 87 × sin 109° = 2550 m^2

Area ΔQSR = ½ × 100 × 103 × sin 74° = 4951 m^2

Hence Area of plot of ground = 7500 m^2 (3 sf)

---

21. Similar to Qu. 13. Use parallel lines etc. to find angles.

∠GAE = 52° - 36° = 16°

Use cosine Rule

$$GE^2 = 200^2 + 160^2 - 2\times200\times160\times\cos 16°$$

Distance between airports = 64 km (2 sf)

Solutions
9 Trigonometry – Sine, Cosine Rule *(continued)*

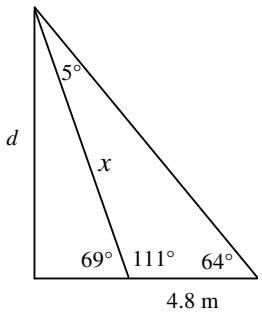
22. Area of triangle = ½ × 7.2 × 10.3 × sin 34°
    = 20.73 m^2
    Area of rectangle = 8.6 × 10.3 = 88.58 m^2
    Total area = 20.73 + 88.58 = 109.31 m^2
    12 litres will cover 12 x 8 = 96 m^2
    No, this is not enough paint.

---

23. Use cosine Rule
    $$PR^2 = 140^2 + 120^2 - 2\times140\times120\times\cos132°$$
    PR = 237.66…
    Hence PR = 238 metres (3 sf)

---

24. Draw diagram and fill in angles
    Use sine rule
    $$\frac{4.8}{\sin 5} = \frac{x}{\sin 64}$$
    x = 49.5 metres
    Use SOH-CAH-TOA
    $$\sin 69 = \frac{d}{49.5}$$



    Hence d = 46.21 metres (now add on height of student)
    Height of building = 46.21 + 1.5 = 47.7 metres (3 sf)

---

25. Area = ½ × 300 × 340 × sin 125 = 41,776.75.. m^2
    = 41,800 m^2 (3 sf)

---

26. Draw the diagram and using alternate angles find that
    ∠ PQR = 40° + 20° = 60°
    Let QR = d
    Using sine rule: $\frac{d}{\sin 85} = \frac{30}{\sin 60}$
    Hence d = 34.509…
    Distance: ship at R to lighthouse Q = 34.5 km (3 sf)

---

27. Use cosine rule
    $$AB^2 = 70^2 + 100^2 - 2\times70\times100\times\cos 65°$$
    AB = 94.7805…
    Hence AB = 95 metres (2 sf)

---

28. Area = ½ × 10 × 12.6 × sin 72° = 59.9165… m^2
    = 59.9 m^2 (3 sf)

---

29. ∠ABP = 40° (angle sum triangle PTB)
    Use sine rule in ΔPAB
    $$\frac{5.6}{\sin 10} = \frac{AP}{\sin 40}$$ hence AP = 20.73 metres
    Now use SOH-CAH-TOA in ΔPTA
    $$\cos 40 = \frac{PT}{AP} = \frac{PT}{20.73}$$ So, PT = 15.88 = 15.9 m (2 sf)

---

30. a) Area = ½ × 6 × 7 × sin 120° = 18.186..… m^2
    = 18 m^2 (2 sf)

    b) Let angle be θ
    For maximum area, sin θ must be a maximum
    Maximum value of sine function is 1
    This occurs when angle is 90°
    Hence θ should be 90° for maximum area.

---

31. Draw diagram and mark in angles – using bearings
    ∠RLT = 15° and ∠ TL West = 30°
    ∠RTL = 30° (alternate angles)
    Now use sine rule
    $$\frac{10}{\sin 30} = \frac{RT}{\sin 15}$$ hence RT = 5.176….
    Ship has travelled 5.2 km (2 sf) from R to T

Solutions

10 Gradient and Straight Line

1. a) Gradient AB = $\frac{3-(-7)}{4-(-1)} \rightarrow \frac{10}{5} \rightarrow 2$

   b) Use y = mx + c Eqn is: y = 2x – 5

   c) (3k, k) lies on AB, so it will satisfy the equation

   Hence, k = 2(3k) – 5 k = 6k – 5 5 = 5k k = 1

---

2. Gradient = $\frac{a-t}{a^2-t^2} = \frac{\cancel{a-t}}{(a+t)\cancel{(a-t)}} = \frac{1}{a+t}$

---

3. a) Gradient AB = $\frac{6-4}{6-2} \rightarrow \frac{2}{4} \rightarrow \frac{1}{2}$

   Use y = mx + c, so y = ½ x + c

   Need to find c, so use point (2, 4)

   4 = ½ (2) + c 4 = 1 + c c = 3

   Equation is y = ½ x + 3

   b) To find M, we know that y = 0

   Hence 0 = ½ x + 3 solving gives x = - 6

---

4. This is a simplified version of Question 3.

---

5. Gradient = $\frac{3-0}{10-4} \rightarrow \frac{3}{6} \rightarrow \frac{1}{2}$

   So, T = ½ S + c

   Find c using (4, 0) in the equation

   0 = ½ (4) + c 0 = 2 + c c = -2

   Equation is: T = ½ S – 2

---

6. Gradient = $\frac{9-1}{4-0} \rightarrow \frac{8}{4} \rightarrow 2$

   y-intercept = 1 Equation is: y = 2x + 1

---

7. Gradient = $\frac{9-3}{3-0} \rightarrow \frac{6}{3} \rightarrow 2$

   y-intercept = 3 Equation is: y = 2x + 3

---

8. Gradient = $\frac{50-5}{60-0} \rightarrow \frac{45}{60} \rightarrow \frac{3}{4}$

   y-intercept = 5 Equation is: y = ¾ x + 5

---

9. a) Draw graph - plot points (0, 10) – initial state and (6, 40) - 6 mins to add 30 litres at 5 litres/min and 40 litres (30 litres added to existing 10)

   b) Gradient = $\frac{40-10}{6-0} \rightarrow \frac{30}{6} \rightarrow 5$

   y-intercept = 10

   Equation is: V = 5x + 10

Applications of straight line

1. B is (12, 40) and A is (0, 4)

   Gradient = $\frac{40-4}{12-0} \rightarrow \frac{36}{12} \rightarrow 3$, y-intercept = 4

   Equation is: m = h + 4

---

2. Gradient = $\frac{100-40}{4-0} \rightarrow \frac{60}{4} \rightarrow 15$, y-intercept = 40

   Equation is: H = 15t + 40

---

3. B is (90, 82) and A is (0, 12)

   Gradient = $\frac{82-12}{90-0} \rightarrow \frac{70}{90} \rightarrow \frac{7}{9}$, y-intercept = 12

   Equation is: $g = \frac{7}{9}h + 12$

---

4. a) Gradient = $\frac{6-2}{12-0} \rightarrow \frac{4}{12} \rightarrow \frac{1}{3}$, y-intercept = 2

   Equation is: $y = \frac{1}{3}x + 2 \rightarrow 3y = x + 6$

   which can be re-arranged to: $3y - x = 6$

   b) Solve simultaneously: $3y - x = 6$ .....(1)
   $4y + 5x = 46$ .....(2)

   multiply (1) by 5 and add giving y = 4

   substitute into (1) giving x = 6

   Co-ordinates are: (6, 4)

---

5. a) Gradient = $\frac{120-160}{12-8} \rightarrow \frac{-40}{4} \rightarrow -10$

   Equation is: P = - 10t + 160 or P = 160 – 10t

   b) Put P = 70

   70 = 160 – 10t and solve for t

   10t = 160 – 70 10t = 90 t = 9

   Expected to be unconscious at 1700 hrs

---

6. Draw graph – plot points (0, 240) and (12, 0)

   Gradient = $\frac{0-240}{12-0} \rightarrow \frac{-240}{12} \rightarrow -20$

   y-intercept = 240

   Hence equation is: V = -20t + 240 or V = 240 – 20t

---

7. Gradient = $\frac{162-138}{80-0} \rightarrow \frac{24}{80} \rightarrow \frac{3}{10}$

   y-intercept = 138

   Hence equation is: $s = \frac{3}{10}t + 138$

Solutions

11 Simultaneous Equations

1. a) Let cost of 1 nights stay = £ n
Let cost of 1 breakfast = £ b

$3n + 2b = 145$ … (1)

b) $5n + 3b = 240$ … (2)

c) Solve simultaneously to find b, eliminate n

(1) × 5 and (2) × 3 then subtract:

b = £5

---

2. a) $9b + 16w = 2520$ … (1)

b) $13b + 12w = 2640$ … (2)

c) Solve to find w and b (1) × 3 and (2) × 4

then subtract to get b = £1.20 and w = 90p

Final design costs $11b + 14w$ = £25.80

---

3. a) $4p + 3g = 130$ … (1)

b) $2p + 4g = 120$ … (2)

c) solve (2) × 2 then subtract: g = 22p, p = 16p

hence, 3 peaches + 2 grapefruit cost: 92 pence

---

4. a) $2x + 3y = 580$ … (1)

b) $x + y = 250$ … (2)

c) eliminate y to find x (2) × 3 and subtract

x = 170 So **170 tickets sold to members**.

---

5. a) $4x + 5y = 1550$ … (1)

b) $2x + 7y = 1450$ … (2)

c) Solve to find x and y (2) × 2 and subtract

to get y = £1.50 and x = £2.00

8 patterned and 1 plain will cost $8x + 1y$ = £17.50

---

6. a) Gradient = $\frac{6-2}{12-0} \rightarrow \frac{4}{12} \rightarrow \frac{1}{3}$, y-intercept = 2

Equation is: $y = \frac{1}{3}x + 2 \rightarrow 3y = x + 6$

which can be re-arranged to: $3y - x = 6$

b) Solve simultaneously: $3y - x = 6$ .....(1)
$4y + 5x = 46$ .....(2)

multiply (1) by 5 and add giving y = 4
substitute into (1) giving x = 6
Co-ordinates are: (6, 4)

---

7. a) $2l + 2b = 260$ … (1)

b) $5l + 8b = 770$ … (2)

c) (1) × 4 then subtract gives:

l = 90cms; b = 40 cms

---

8. a) Cost of 2 children (13 & 15) = 2x
Cost of 3 children (under 10) = 3y
Cost of adult = £8
Total paid = £19

Hence: $2x + 3y + 8 = 19$ or $2x + 3y = 11$

b) $4x + y + 8 = 15$ or $4x + y = 7$

c) Solve simultaneously:

$2x + 3y = 11$ …(1)

$4x + y = 7$ …(2)

(1) × 2 and subtract, giving $y = 3$ and $x = 1$

(i) single ticket for 14 year old = £ 1

(ii) single ticket for 7 year old child = £ 3

---

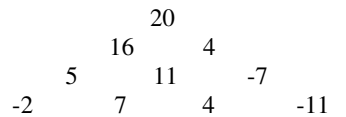
9. a) $3x + 2y = 38$ … (1)

b) $2x + 5y = 51$ … (2)

(1) × 2 and (2) × 3 and subtract: y = 7 and x = 8

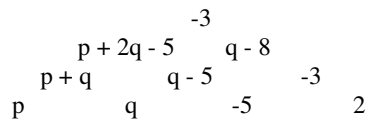
Ht. cylinder = 8 cm, Ht. cuboid = 7 cm.

10.



a) number on shaded brick is 20

b)



So, from diagram: p + 2q -5 + q – 8 = -3
or p + 3q = 10

c) Using the same idea, 2q – p = 5

Solve simultaneously:
p + 3q = 10 … (1)
– p + 2q = 5 ….(2)

adding: q = 3, p = 1

---

11. a) 3x + 4y = 65

b) 5x + 7y = 112

c) Solve simultaneously: x = 7; y = 11

---

12. a) 25 tiles

b) Form two equations – using 1st & 2nd arrangements
1 = 2 + a + b a + b = -1
5 = 8 + 2a + b 2a + b = -3
Solve to get a = -2, b = 1

---

13. a) 3.5 and 4.6

b) Using 1st & 2nd rods 1.1 = A + b
1.4 = A + 4b
solving gives: b = 0.1 and A = 1

$$h = 1 + 0.1n^2$$

Solutions
12 Functions
Properties of the Parabola

1. a) a = -1, b = 3 (roots of the equation)

   b) The point (0, -6) lies on the curve, so it will satisfy the equation of the curve.
   Hence, $-6 = k(0 + 1)(0 - 3)$ so, $-6 = -3k$
   $k = 2$

   c) Axis of symmetry is x = 1.
   When x = 1, y = 2(1 + 1)(1 – 3)
   Hence y = -8

---

2. a) Cuts y axis when x = 0, so y = -12

   b) B and C are roots of eqn. $x^2 + x - 12 = 0$

   factorise $(x+4)(x-3) = 0$ hence x = -4, or 3

   B is (4, 0) and C is (3, 0)

   c) Axis of symmetry is x = - ½ .

   When x = - ½ , $y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 12$

   Hence $y = \frac{1}{4} - \frac{1}{2} - 12 \rightarrow -12\frac{1}{4}$

   Co-ords of min t.p. are (- ½ -12 ¼)

---

3. Use the formula with a = 3, b = 7, c = -2

   x = -0.21 or x = - 2.12

---

4. a) When x = 0, y = -3

   b) Solve the equation by factorisation

   $4x^2 + 4x - 3 = 0 \quad (2x-1)(2x+3) = 0$

   hence x = ½ or $x = -\frac{3}{2}$

   c) axis of symmetry is x = - ½
   when x = - ½ y = -4 co-ords of min t.p. (- ½ , -4)

Applications of the parabola

1. a) Area of glass = (7 – 2x)(10 – 2x)

   A = 70 – 14x – 20x + $4x^2$ Hence, **A = $4x^2$ – 34x + 70**

   b) 28 = $4x^2$ – 34x + 70

   Re-arrange: $4x^2$ – 34x + 42 = 0

   Divide by 2: $2x^2$ – 17x + 21 = 0

   Factorise: (2x - 3)(x - 7) = 0

   Hence x = 1 ½ or x = 7

   x cannot = 7, since this is width of frame,

   So x = 1 ½ cms

---

2. a) Area of A is: (x + 6)(x – 1)

   Area of B is: 3(x + 3)

   b) So, (x + 6)(x – 1) = 3(x + 3)

   Hence, $x^2 + 6x - x - 6 = 3x + 9$

   simplify: $x^2 + 2x - 15 = 0$

   factorise: $(x+5)(x-3) = 0$

   so, x = 3 or x = -5. x = -5 is not possible

   Hence x = 3

---

3. a) Solve the quadratic by factorisation

   $8 + 2x - x^2 = 0$ Hence, $(4-x)(2+x) = 0$

   x = 4 or x = -2, so F is (4, 0)

   The fly is 4 feet to the right of the snake.

   b) axis of symmetry is when x = 1

   Hence max height is H = 8 + 2 – 1 = 9 feet.

---

4. $H(3) = 9 + 6(3) - 3(3)^2 = 0$

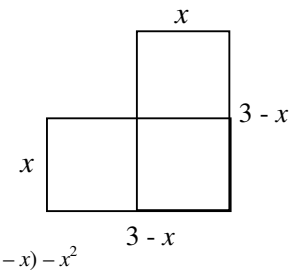
   This indicates that the shell is now level with the cliff again.

---

5. a) Since BC = CD then

   2BC + 2x = 6 → BC + x = 3 So, BC = 3 – x

   b) Area of rectangle
   = x (3 – x)
   There are 2 rectangles
   but then we have counted
   the square twice.



   Hence Area = $x(3-x) + x(3-x) - x^2$
   A = 3 x – x^2 + 3 x – x^2 – x^2
   A = 6x – 3x^2

   c) Find the roots of the equation 6x – 3x^2 = 0
   Factorise: 3x (2 – x) = 0
   Hence x = 0 or x = 2 x = 0 is not possible, So x = 2.
   Axis of symmetry is x = 1
   Max value is on axis of symmetry: A = 6 – 3 = 3 m^2

---

6. a) $l = w + 2$

   b) Area of extension is : $w(w + 2)$ → $w^2 + 2w$
   This must not be more than 40% original size
   $120 \times 0.4 = 48$ So $w^2 + 2w = 48$ (largest extension)
   Hence $w^2 + 2w - 48 = 0$ so (w – 6)(w + 8) = 0
   w = 6 or -8 (not possible) Width = 6, Length = 8

---

7. a) 18 – 2x cms

   b) V = $x(18 - 2x) \times 100$ V = 1800x – 200x^2

   c) Put 1800x – 200x^2 = 0 and solve equation by factorising: 200x(9 - x) = 0 x = 0 or x = 9
   maximum is on axis of symmetry x = 4 ½
   dimensions of gutter are 9 cm wide × 4 ½ cm high

Solutions
13 Making & Using Formulae

1. DB = $2x$ Let TD = h cms
Area TDB = ½ base × height = ½ × h × $2x$ = hx
Area of clipboard = $3x \times 4x = 12x^2$
Area triangle = ¼ area clipboard
$hx = 3x^2$ so, $h = 3x$

---

2. Put d = 20 into formula
$$20 = \frac{n(n-3)}{2} \rightarrow 40 = n^2 - 3n$$
Re-arrange : $n^2 - 3n - 40 = 0$ factorise to solve
$(n-8)(n+5) = 0$ so $n = 8$ or $n = -5$
number of sides must be 8 (-5 not possible)

---

3. a) $3 \times 25 + 5 \times 3 = 90$ pence
b) $75 + (m - 3) \times 5 \rightarrow 75 + 5m - 15 \rightarrow 60 + 5m$
c) $80 + (m - 2) \times 2 \rightarrow 80 + 2m - 4 \rightarrow 76 + 2m$
$76 + 2m < 60 + 5m$
$16 < 3m$ $m > 5.33$ mins
Minimum number of minutes = 6 minutes

---

4. a) put c = 3 $I = \frac{20}{2^3} \rightarrow \frac{20}{8} = 2.5$
b) put I = 10
$$10 = \frac{20}{2^c} \rightarrow 2^c = \frac{20}{10} \rightarrow 2^c = 2 \text{ So, c = 1}$$
c) max intensity 2^c is smallest i.e. when c = 0
then $2^0 = 1$ max intensity is 20

---

5. a) $30 + x$
b) Area = $(30 + x)(20 + x) \rightarrow 600 + 30x + 20x + x^2$
Area = $600 + 50x + x^2$
c) New Area = $30 \times 20 \times 1.4 = 840$
Solve equation: $840 = 600 + 50x + x^2$
re-arrange: $x^2 + 50x - 240 = 0$
Use formula with a = 1, b = 50 c = -240
$x = 4.41$ cm or $x = -54.41$ cms
Hence min dimensions are: 35 cms by 25 cms
(nearest cm – remember dimensions are minimum)

---

6. Volume of space = Vol. Cylinder – Vol cuboid
Calculate volume of cylinder.
$$V = \pi r^2 h \rightarrow V = \pi \times 6^2 \times 20 = 720\pi \text{ cm}^3$$
Cross section of cuboid is a square

Diagonal = 12 cm, Area = ½ diag x diag
Area = ½ x 12 x 12 = 72 cm^2

Or use Pythagoras
Side of square = $\sqrt{6^2 + 6^2} = \sqrt{72}$ cm
Area of square -= $\sqrt{72} \times \sqrt{72} = 72$ cm^2

Volume of cuboid = 72 x 20 = 1440

Hence vol of space = $720\pi - 1440 = 720(\pi - 2)$

---

7. a) Base rate = £425 per person
2 extra adults so less £ 60 per person
Hence cost : 365×4 = £ 1460
b) Base rate = £425 per person for P persons
(P – 2) extra adults,
so reduction is: £ $30 \times (P - 2)$ per person
Hence cost : $425 - 30(P - 2)$ per person
For P persons: Cost = £ $P[425 - 30(P - 2)]$

---

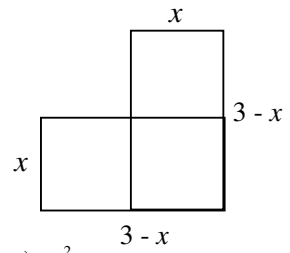
8. Supplement of y is 180° - y
Angles in triangle add up to 180°
So, a + b + 180 – y = 180
Hence, y = a + b

---

9. a) (42 – 15) charged at 35p per min = £9.45
= rental £ 17.50, Total cost = £ 26.95
b) $(t - 15) \times 0.35$ for calls + rental of £17.50
$\rightarrow 0.35t - 5.25 + 17.5 \rightarrow$ **£ 12.25 + 0.35t**

---

10. a) Since BC = CD then
$2BC + 2x = 6 \rightarrow BC + x = 3$ So, $BC = 3 - x$
b) Area of rectangle
$= x(3 - x)$
There are 2 rectangles
but then we have counted
the square twice.



Hence Area = $x(3 - x) + x(3 - x) - x^2$
$A = 3x - x^2 + 3x - x^2 - x^2$
$A = 6x - 3x^2$
c) Find the roots of the equation $6x - 3x^2 = 0$
Factorise: $3x(2 - x) = 0$
Hence $x = 0$ or $x = 2$ x = 0 is not possible, So $x = 2$.
Axis of symmetry is x = 1
Max value is on axis of symmetry: A = 6 – 3 = 3 m^2

---

11. a) Pupils 12 x £4.50 = £54
Adult: 1 free so 2 x £7.00 = £14.00
Total cost = £68.00
b) Cost of pupils: £ $4 \times p$
Adults: 2 free d – 2 adults pay £ 6(d – 2)
Total cost: £ 4p + 6(d – 2)

---

12. Substitute N = 26 into formula
$$26 = \frac{30v}{2+v} \text{ re-arrange } 26(2+v) = 30v$$
Hence, $52 + 26v = 30v \rightarrow 4v = 52 \quad v = 13$
Speed of cars = 13 metres per second.

---

13. a) C = 15d
b) C = 50 + 10d
c) Eurocar: 170 = 15d d = 170 ÷ 15 d = 11.333…
Apex: 170 = 50 + 10d 10d = 120 d = 12
Apex will give them 12 days.
(NB could also hire car for 3 days from Eurocar with deposit)

Solutions

13 Making & Using Formulae *(continued)*

14. $s = \dfrac{8.6+7.4+10+9.1}{2} = 17.55$

$A = \sqrt{(17.55-8.6)(17.55-7.4)(17.55-10)(17.55-9.1)}$

$A = \sqrt{8.95 \times 10.15 \times 7.55 \times 8.45} = \sqrt{5795.524} = 76.128..$

A = 76 cm^2 (2 sf)

---

15. a) Expenses: $250 \times 0.29 + 300 \times 0.15 = £117.50$

b) $E = 250 \times 0.29 + (t - 250) \times 0.15$

$E = 72.50 + 0.15t - 37.50 = 35 + 0.15t$

---

16. a) 16

b) i) $1300 \div 150 = 8.66..$ Integral part = 8

ii) $\left[\dfrac{1300}{B}\right] \times \left[\dfrac{1000}{L}\right]$

---

17. a) $18 - 2x$ cms

b) $V = x(18 - 2x) \times 100$ $V = 1800x - 200x^2$

c) Put $1800x - 200x^2 = 0$ and solve equation by factorising: $200x(9 - x) = 0$ $x = 0$ or $x = 9$

maximum is on axis of symmetry $x = 4\frac{1}{2}$

dimensions of gutter are 9 cm wide × 4 ½ cm high

---

18. a) Cost = £13.50 + £0.75 × 4 = £ 16.50

b) $C = 13.50 + 0.75 \times (w - 10)$

$C = 13.50 + 0.75w - 7.5 = 6 + 0.75w$

---

19. Ellipse will cut x-axis at -6 and 6

and y-axis at 4 and – 4

(Look for the patterns in the formulae

– denominators are squares of where it cuts the axes.)

---

20. a) Use formula

$$P = \frac{(40+15)(40-15+1)}{2} \rightarrow \frac{55 \times 26}{2} \rightarrow 715$$

b) There are a on the top row and 2a on the bottom row, so put b = 2a in formula

$$P = \frac{(2a+a)(2a-a+1)}{2} \rightarrow \frac{(3a)(a+1)}{2} \rightarrow \frac{3a^2+3a}{2}$$

c) Can P = 975 with whole number solutions ?

$975 = \dfrac{3a^2+3a}{2} \rightarrow 1950 = 3a^2 + 3a$

$3a^2 + 3a - 1950 = 0$ dividing by 3

$a^2 + a - 630 = 0$

Try solving with the formula

with a = 1, b = 1, c = -630

find that solution involves √2521 = 50.2…

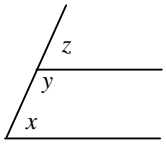
So no whole number solutions

---

21. a) angle above b is 72° (corresponding)

hence b = 180 – 72 = 108°

b) In the diagram



$\angle x = \angle z$ (corresponding)

$\angle y + \angle z = 180°$ (supplementary)

hence $\angle y + \angle x = 180°$

---

22. a) Adults: £555 Child: FREE Extra nights: £29 × 3

Total cost = £ 642

b) $C = 555 + 29(t - 14) \rightarrow C = 555 + 29t - 406$

$C = 149 + 29t$

---

23. a) Area of border = $x^2 - y^2 = 48$

Hence $(x - y)(x + y) = 48$

b) Factors of 48 are

48 × 1, 24 × 2, 16 × 3, 12 × 4, 8 × 6

Since x and y are greater than 10, then x + y > 20

so only need to consider 48 × 1 and 24 × 2

hence $x + y = 48$ and $x - y = 1$

no solution – numbers need to differ by 1

so one must be odd, and 48 is even

or $x + y = 24$ and $x - y = 2$

so, $x = 13$, $y = 11$

Hence: $x = 13$, $y = 11$

---

24. a) If AB = 2 then BC = 2 (it is a square)

By Pythagoras: $AC^2 = 2^2 + 2^2$ so AC = $\sqrt{8} \rightarrow 2\sqrt{2}$

b) In any square of side a.

Diagonal = $\sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

Ratio of side to diagonal is: a : a√2

which is 1 : √2

---

25. a) i) $12h$ minutes

ii) $\dfrac{v}{100} \times 10 \rightarrow \dfrac{v}{10}$ minutes

iii) $\dfrac{12h}{60} + \dfrac{v}{600} \rightarrow \dfrac{120h}{600} + \dfrac{v}{600} \rightarrow T = \dfrac{120h+v}{600}$

b) $T = \dfrac{120 \times 8 + 900}{600} \rightarrow \dfrac{1860}{600} \rightarrow 3.1$ hours

Total time = 3.1 hours each way = 6.2 hours

No, it should not be started

Since 1300 to 1900 is only 6 hours.

---

27. a) £5 + £26 + £18 = £49

b) Total Fee payable = 5 + 26 + (P – 15)

= 31 + P – 15 = 16 + P

---

26. a) Plot graph when t = 0, M = 80 (0, 80)

(1, 40), (2, 20), (3, 10), (4, 5)

b) $\dfrac{5}{8} = 80(2)^{-t} \rightarrow \dfrac{5}{8} = \dfrac{80}{2^t} \rightarrow 2^t = \dfrac{640}{5} = 128$

$2^t = 2^7$ hence t = 7. **It will take 7 years.**

Solutions

14 Trig Graphs & Equations

1. hypotenuse = 5 (Pythagoras – or 3,4,5 triangle)

   $\sin x = \frac{3}{5} \quad \cos x = \frac{4}{5}$

   $\sin^2 x + \cos^2 x = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$

---

2. *a* is the amplitude, using symmetry, top of wave is 5
   Hence a = -5 (since sine wave is inverted)

   b is number of waves in 360°
   whole wave will take up 120°
   so *b* = 3

   a = -5, *b* = 3

---

3. a) 1.30 pm is 1.5 hours after midnight,
   put t = 1.5 into the formula

   $D = 12.5 + 9.5\sin(30 \times 1.5)$

   Depth = 19.217… = 19.2 metres (1 dp)

   b) Maximum depth is when sine is maximum (= 1)
   Max depth = 12.5 + 9.5 = 22 metres

   Minimum depth is when sine is minimum (= -1)
   Min depth = 12.5 - 9.5 = 3 metres

   Maximum difference is 22 – 3 = 19 metres.

---

4. $y = k\sin ax°$ *k* = 3 (amplitude)
   a = 2 (number of waves in 360°)

---

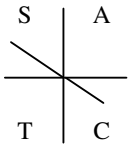
5. $y = a\cos bx°$ *a* = 3 (amplitude)
   b = 2 (number of waves in 360°)

Solving Equations

1. $3\tan x + 5 = 0 \rightarrow \tan x = -\frac{5}{3}$

   $x = \tan^{-1}\frac{5}{3}$ acute $x = 59.03...°$

   $x = 180 - 59 = 121°$ *or* $x = 360 - 59 = 301°$



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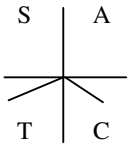
2. $2 + 3\sin x = 0 \rightarrow \sin x = -\frac{2}{3}$

   $x = \sin^{-1}\frac{2}{3}$ acute $x = 41.81..°$

   $x = 180 + 42 = 222°$ *or* $x = 360 - 42 = 318°$



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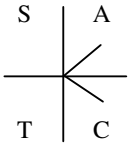
3. $7\cos x - 2 = 0 \rightarrow \cos x = \frac{2}{7}$

   $x = \cos^{-1}\frac{2}{7}$ acute $x = 73.398..°$

   $x = 73°$ *or* $x = 360 - 73 = 287°$



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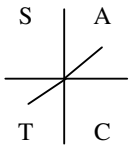
4. $5\tan x - 9 = 0 \rightarrow \tan x = \frac{9}{5}$

   $x = \tan^{-1}\frac{9}{5}$ acute $x = 60.945..°$

   $x = 61°$ *or* $x = 180 + 61 = 241°$



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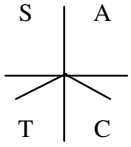
5. $5\sin x + 2 = 0 \rightarrow \sin x = -\frac{2}{5}$

   $x = \sin^{-1}\frac{2}{5}$ acute $x = 23.578..°$

   $x = 180 + 24 = 204°$ *or* $x = 360 - 24 = 336°$



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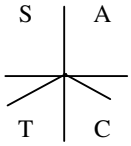
6. $\tan 40 = 2\sin x + 1 \rightarrow \sin x = -\frac{0.1609}{2}$

   $x = \sin^{-1}\frac{0.1609}{2}$ acute $x = 4.614..°$

   $x = 180 + 5 = 185°$ *or* $x = 360 - 5 = 355°$



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7. $2\tan 24 = \tan q \rightarrow \tan q = 0.8905$

   $x = \tan^{-1} 0.8905$ acute $x = 41.683..°$

   q is an acute angle, so q = 42° (2 sf)

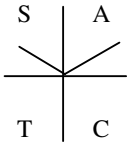
---

8. Solve $y = \sin x$ and $y = 0.4 \rightarrow \sin x = 0.4$

   $x = \sin^{-1} 0.4$ acute $x = 23.578..°$

   $x = 24°$ *or* $x = 180 - 24 = 156°$

   A is (24°, 0.4) and B is (156°, 0.4)



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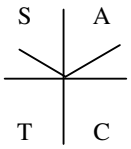
9. a) amplitude = 3, so a = 3
   since max is at 90°, there is 1 wave in 360°
   hence b = 1

   b) $3\sin x = 2 \rightarrow \sin x = \frac{2}{3}$

   $x = \sin^{-1}\frac{2}{3}$ acute $x = 41.81..°$

   $x = 42°$ *or* $x = 180 - 42 = 138°$

   P is (42°, 2) and Q is (138°, 2)



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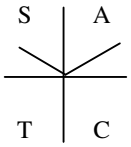
10. a) S is (90°, 1)

    b) $\sin x = 0.5$

    $x = \sin^{-1} 0.5$ acute $x = 30°$

    $x = 30°$ *or* $x = 180 - 30 = 150°$

    T is (30°, 0.5) and P is (150°, 0.5)



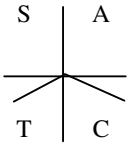
Solutions

14 Trig Graphs & Equations *(continued)*

11. a) A is (90°, 0)

b) $\cos x = -0.5$

$x = \cos^{-1} 0.5$ acute $x = 60°$



$x = 180 + 60 = 240°$ *or* $x = 360 - 60 = 300°$

B is (240°, -0.5) and C is (300°, -0.5)

---

12. a) Maximum value of H is when cosine is maximum (= 1)

h = 1.9 + 0.3 = 2.2 metres

b) After 8 seconds

$h = 1.9 + 0.3\cos(30 \times 8) \rightarrow 1.75$ metres

c) put h = 2.05 in equation

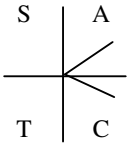
$2.05 = 1.9 + 0.3\cos(30t) \rightarrow 0.15 = 0.3\cos 30t$

$\rightarrow \cos 30t = 0.5 \rightarrow 30t = \cos^{-1} 0.5$

acute $30t = 60°$, 300, …..

so, 30t = 60

t = 2 seconds – first time



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13. a) Put t = 10 for October into formula.

$V = 1 + 0.5\cos(30 \times 10) \rightarrow 1.25$ million gallons

b) t can only take whole number values of 1 to 12

$V(1) = 1 + 0.5\cos(30 \times 1) \rightarrow 1.43$

$V(2) = 1 + 0.5\cos(30 \times 2) \rightarrow 1.25$

$V(3) = 1 + 0.5\cos(30 \times 3) \rightarrow 1$

$V(4) = 1 + 0.5\cos(30 \times 4) \rightarrow 0.75$

$V(5) = 1 + 0.5\cos(30 \times 5) \rightarrow 0.567$

$V(6) = 1 + 0.5\cos(30 \times 6) \rightarrow 0.5$

$V(7) = 1 + 0.5\cos(30 \times 7) \rightarrow 0.567$

$V(8) = 1 + 0.5\cos(30 \times 8) \rightarrow 0.75$

$V(9) = 1 + 0.5\cos(30 \times 9) \rightarrow 1$

$V(10)$ OK see part (a)

$V(11) = 1 + 0.5\cos(30 \times 11) \rightarrow 1.433$

$V(12) = 1 + 0.5\cos(30 \times 12) \rightarrow 1.5$

Council will need to consider water rationing in June.

See next column for an alternative solution:

Alternative solution: (Fuller understanding required)

For t = 1, 2, 3 and 10, 11, 12 the cosine is positive (in 1st and 4th quadrants)

For t = 4, 5, 6 and 7, 8, 9 the cosine is negative.

the minimum value will occur when cos = -1
i..e cos 180°, then t = 6.

Hence V = 1 – 0.5 = 0.5

So, rationing needs to be considered in June

Now look at t = 5 and t = 7,
Work out V for these and you find
For May (t = 5) V= 0.567
For July (t = 7) V= 0.567 (symmetrical)

These are over critical level of 0.55 million gallons

So rationing only needs to be considered in June.

Solutions

15 Ratio & Proportion

1. a) **Ratio -**

	Parents		Teachers		Pupils
	1	:	3	:	15
For pupils					**45**
All quantities × 3	3	:	9	:	**45**

So, 9 teachers must accompany them.

b) 100 Tickets given.
For each 15 pupils,
there must be 3 teachers and 1 Parent

This makes a group of 19.
They can only go in multiples of 19

Largest number of 19s in 100 is 5
Since $5 \times 19 = 95$

So, $5 \times 15 = 75$ pupils can go.

---

2. Brazilian : Columbian

2 : 3

20 kg of Brazilian, would require 30 kg of Columbian coffee, there is not enough Columbian coffee, so we need to see how much an be made with the Columbian coffee

Each 1 kg tin contains

400 gm Brazilian : 600 gm Columbian

So 25 kg =25 000 gm

25 000 ÷ 600 = 41.667 ….. tins

Hence 41 one kg tins can be made

******** Misprint in question

should read: $\frac{f_2}{f_1} = \frac{3}{2}$ and $f_4 : f_3 = 4:3$

3. a) $f_2 = f_1 \times \frac{3}{2}$

and $f_3 = f_2 \times \frac{4}{3}$

So $f_3 = f_2 \times \frac{4}{3} \rightarrow f_3 = \left(f_1 \times \frac{3}{2} \right) \times \frac{4}{3}$

Thus: $\rightarrow f_3 = f_1 \times \frac{12}{6} \rightarrow f_3 = 2f_1$

and $\rightarrow \frac{f_3}{f_1} = \frac{2}{1}$ or $f_3 : f_1 = 2:1$

b) $\frac{f_2}{f_1} = \frac{6}{5}$ *and* $\frac{f_3}{f_1} = \frac{3}{2}$

We want to find $\frac{f_3}{f_2}$

So, $f_3 = \frac{3}{2} \times f_1$

$f_2 = \frac{6}{5} \times f_1 \rightarrow f_1 = \frac{5}{6} \times f_2$

Hence $f_3 = \frac{3}{2} \times f_1 \rightarrow f_3 = \frac{3}{2} \times \left(\frac{5}{6} \times f_2 \right)$

$\rightarrow f_3 = \frac{15}{12} \times f_2 \rightarrow f_3 = \frac{5}{4} \times f_2$

$\rightarrow \frac{f_3}{f_2} = \frac{5}{4}$ *or* $f_3 : f_2 = 5:4$

So the frequency ratio of a major third is 5 : 4

Solutions
16 Variation & Proportion

1. a) $T = \dfrac{kv^2}{r}$

b) If v is multiplied by 3 then v^2 in the formula causes T to be multiplied by 9

If r is halved then T is doubled.

So overall effect is to multiply T by 18

---

2. a) $R = \dfrac{kL}{d^2}$

b) Wire A: $R = \dfrac{3k}{2^2}$ Wire B: $R = \dfrac{kL}{3^2}$

Since resistance is same for both wires $\dfrac{kL}{3^2} = \dfrac{3k}{2^2}$

so, $\dfrac{\cancel{k}L}{3^2} = \dfrac{3\cancel{k}}{2^2} \rightarrow L = \dfrac{3\times 3^2}{2^2} = \dfrac{27}{4} = 6.75\ m$

Length of wire B is 6.75 metres.

---

3. a) $F = \dfrac{kV^2}{R}$

b) If V is multiplied by 2,
then V^2 will cause F to be multiplied by 4

So the frictional force will be 80 kilonewtons

---

4. a) The ratio $\dfrac{d}{t^2} = 4$ for all entries in the table

i.e. $d = 4t^2$ This is direct proportion

b) $d = 4t^2$

c) When time is multiplied by 6, d is multiplied by 6^2 so d is multiplied by 36.

---

5. a) $T = \dfrac{kS}{E}$

b) T = 12 , when S = 20 000 and E = 20

$12 = \dfrac{20000k}{20} \rightarrow 12 = 1000k \rightarrow k = \dfrac{12}{1000}$

c) $T = \dfrac{12}{1000} \times \dfrac{36000}{24} \rightarrow \dfrac{36}{2} \rightarrow 18$ minutes

---

6. a) $L = kD\sqrt{S}$

b) L = 30, when D = 550 and S = 81

$30 = k \times 550\sqrt{81} \rightarrow k = \dfrac{30}{550\times 9} = \dfrac{1}{165}$

c) $L = \dfrac{1}{165} \times 693 \times \sqrt{100} = 42$ litres

---

7. $A = kD^2$

For Moon: surface area = 3.8×10^7

If diameter is multiplied by 2 then A will be $\times 4$

Hence surface area of planet =

$4\times 3.8\times 10^7 = 1.52\times 10^8\ km^2$

---

8. a) For each x, y in the table the product xy is constant, Hence x and y are in inverse proportion.

b) $xy = 9$ *or* $y = \dfrac{9}{x}$

---

9. a) $N = \dfrac{k}{s^2}$

b) If s is doubled then this will halve N, BUT since

s is squared, the result will be $\left(\dfrac{1}{2}\right)^2$ or $\dfrac{1}{4}$

So only ¼ the number of letters on the page.

---

10. a) $T = \dfrac{kL}{\sqrt{H}}$

b) T = 10, when L = 3.75 and H = 2.25

$10 = \dfrac{k\times 3.75}{\sqrt{2.25}} \rightarrow k = \dfrac{10\times\sqrt{2.25}}{3.75} = 4$

$T = \dfrac{4\times 5}{\sqrt{2.56}} \rightarrow 12.5$ seconds

---

11. $P = kV^3$

P = 75, when V = 4

$75 = k\times 4^3 \rightarrow k = \dfrac{75}{64}$

When wind speed doubled, V = 8

$P = \dfrac{75}{64}\times 8^3 \rightarrow \dfrac{75\times 8\times 64}{64} = 600$ watts

Alternative way:

$P = kV^3$ if V is doubled,

then P will be multiplied by 2^3 or 8

Hence Power will be 75 x 8 = 600 watts.

Solutions
17 Distance, Speed, Time

Calculations

This question contained a misprint by the SQA, the units should be miles not kilometres.

1. a) $T = \frac{D}{S}$ $\quad T = \frac{x}{75}$

   b) Average speed = Total Distance ÷ Total Time

   Total Distance = 2x

   Total Time = $\frac{x}{75} + \frac{x}{50} \rightarrow \frac{2x}{150} + \frac{3x}{150}$

   $= \frac{5x}{150} \rightarrow \frac{x}{30}$

   Average speed = $2x \div \frac{x}{30} \rightarrow \frac{2x}{1} \times \frac{30}{x} = 60\ mph$

---

2. S = D ÷ T Time = 88 × 24 hours = 2112 hours

   Distance = circumference = $\pi \times 1.2 \times 10^7$

   Hence speed = $\pi \times 1.2 \times 10^7 \div 2112 = 17849.95\ldots$

   Speed = 18000 km per hour (2 sf)

---

3. T = D ÷ S

   T = $5.9 \times 10^9 \div 3.0 \times 10^5$ seconds

   T = $1.9666.. \times 10^4$ secs

   Change to hours ÷ 3600

   T = 19666.667 ÷ 3600 = 5.46296… hours

   T = 5 ½ hours (to nearest ½ hour)

---

4. T = D ÷ S

   T = $2.3 \times 10^8 \div 3.0 \times 10^5$ seconds

   T = 766.666… secs

   Change to minutes ÷ 60

   T = 766.667 ÷ 60 = 12.7777… mins

   T = 13 minutes (to nearest minute)

---

5. a) The trunk road is from 0900 to 0915

   Distance = 6 miles Time = 15 minutes

   Average Speed = D ÷ T = 6 ÷ 0.25 = 24 mph

   b) Between 0915 and 0925 she joins the motorway and is accelerating.

   c) Calculate average speed on motorway, for straight line section of graph from
   0925 to 0935 : 16 miles in 10 minutes = 96 mph
   or
   0920 to 0935 : 22 miles in 15 minutes = 88 mph

   Jennifer appears to have broken the speed limit on the motorway.

Graphs & Interpretation

1. a) X jumped first

   b) X did not open parachute immediately after jumping because his rate of descent was higher than Y's. His graph is a lot steeper, showing he lost height more rapidly just after he jumped.

---

2. a) A, B, H, J

   b) There will be dips for each corner C, F, G
   The initial acceleration will be longer from A to C
   There will be one longer horizontal at the beginning (A to C), then three short sections (CF, FG, GJ)

---

3. a) CD is when the driver put petrol into the tank.

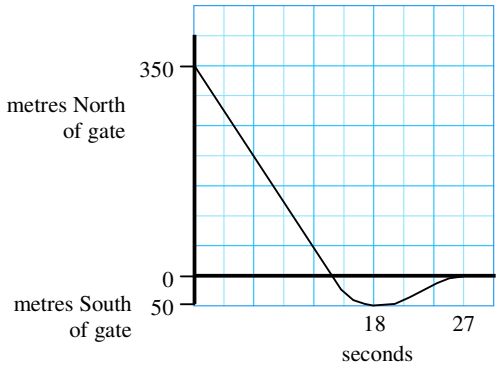
   b) BC and DE show motorway driving.
   The slope of the graph is less, because less petrol is consumed for a given distance.

---

4. a) **Initially the motor cycle is in front of the car, travelling faster**, the motor cycle slows down and the car accelerates, until the **car passes the motor cycle at point A**.
   The motor cycle reaches the end of the road 6 seconds after the car.

   b) Initially the bus is stopped 300m north of the gate. It sets off, accelerating and then slowing down to arrive at the bus stop opposite the gate 30 seconds later.

   c) The graph provided in the booklet is not correct, since you need to be able to show 50m South of the gate for the roundabout



Solutions
18 Sequences

1 a) $S_3 = 1+3+5 = 9$

b) Look at S_2 and S_4

$S_2 = 1+3 = 4 = 2^2$

$S_4 = 1+3+5+7 = 16 = 4^2$

Hence, $S_n = n^2$

c) $(n+1)^{th}\ term = S_{n+1} - S_n$

(Try it with some of the terms to convince yourself)

$S_{n+1} - S_n = (n+1)^2 - n^2$

$\rightarrow n^2 + 2n + 1 - n^2 \rightarrow 2n+1$

---

2. a) 2p – 4, 2p – 2, 2p, 2p + 2, 2p + 4, 2p + 6, 2p + 8

b) Mean = sum divided by 7
Sum = 14p +14
Divide by 7 gives 2p + 2 → 2(p + 1)

---

3. a) $2^n = 32 \quad 32\ is\ 2^5, \quad so, \quad 2^n = 2^5$
hence n = 5

b) Sum of five numbers = (1 + 2 + 4 + 8 + 16) = 32 – 1

c) These are powers of 2
sum of 2 numbers is $2^2 - 1$
sum of 3 numbers is $2^3 - 1$
sum of 4 numbers is $2^4 - 1$
Hence sum of n numbers is $2^n - 1$

---

4. a) 5th pattern is:
$2 \times (1 + 2 + 3 + 4 + 5) - 5 = 25$

b) n^{th} pattern is
$2 \times (1 + 2 + 3 + \ldots.. + n) - n = n^2$
since the patterns gives squares

c) If: $2 \times (1 + 2 + 3 + \ldots.. + t) - t = 289$
then comparing with b) we see that $t^2 = 289$
hence $t = \sqrt{289} = 17$

---

5. a) $7^3 + 1 = (7+1)(7^2 - 7 + 1)$

b) $n^3 + 1 = (n+1)(n^2 - n + 1)$

c) Re-arrange this as follows:

$8p^3 + 1 = 8(p^3 + 1) - 7$

$p^3 + 1 = (p+1)(p^2 - p + 1)$

$8p^3 + 1 = 8(p+1)(p^2 - p + 1) - 7$

---

6. a) $7 = 4^2 - 3^2$

b) $19 = 10^2 - 9^2$

c) n^{th} odd number $= n^2 - (n-1)^2$

$n^2 - (n-1)^2 \rightarrow n^2 - n^2 + 2n - 1 \rightarrow 2n - 1$

d) Let odd number be 2n – 1
Hence next consecutive odd number is 2n + 1
Product is: $(2n-1)(2n+1) \rightarrow 4n^2 - 1$
Since $4n^2$ is even then $4n^2 + 1$ is odd

---

7. a) $12 + 5 - 7$
$15 + 6 - 9$

b) n^{th} term: $3n + (n+1) - (2n-1)$

$3n + n + 1 - 2n + 1 \rightarrow 2n + 2$

---

8. a) $24^2 - 23^2 = 24 + 23 = 47$

b) $(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 \rightarrow 2n + 1$

---

9. a) $5, -1, 4, \quad 3, \quad 7, \quad 10$

b) Sum of first 6 terms: -4
Four times fifth term: $4 \times -1 = -4$

c) $p,\ q,\ p+q,\ p+2q,\ 2p+3q,\ 3p+5q$
Sum of first 6 terms: $8p + 12q$
Four times fifth term:

$$4 \times (2p + 3q) \rightarrow 8p + 12q$$

---

10. a) 1 + 5 + 9 + 13 + 17 + 21 + 25 + 29 + 33 = 9 × 17

b) $n \times (2n - 1)$

---

11. a) (20 x 8) – (22 x 6) = 160 – 132 = 28

b) Let upper left corner be n
then upper right corner is n + 2
lower left corner is n + 14
lower right corner is n + 2 + 14
Hence:

$(n+14)(n+2) - (n+16)(n)$

$= n^2 + 2n + 14n + 28 - n^2 - 16n \rightarrow 28$

So very 3 x 3 square will give the answer 28

---

12. a) $\dfrac{7^2 \times 8^2}{4}$

b) $\dfrac{n^2 \times (n+1)^2}{4}$

c) This will be sum of 1st n consecutive cubes, minus the first 7 consecutive cubes.

$$\frac{n^2 \times (n+1)^2}{4} - \frac{7^2 \times 8^2}{4}$$

---

13. a) 12×14

b) $\frac{1}{2}(n-1) \times \frac{1}{2}(n+3)$

$\rightarrow \frac{1}{4}(n-1)(n+3)$

---

14. a) Fairly self evident – extend to lower diagonals and complete the shell.

b) If N = 1, then $1 = a - b$

If N = 2, then $5 = 4a - 2b$

Solve simultaneously to get: $a = \frac{3}{2}, \quad b = \frac{1}{2}$

---

15. a) $y^3 + (4+5+6)y^2 + (4\times5 + 4\times6 + 5\times6)y + 4\times5\times6$

$y^3 + 15y^2 + 74y + 120$

b) $y^3 + (a+b+c)y^2 + (a\times b + a\times c + b\times c)y + a\times b\times c$

$y^3 + (a+b+c)y^2 + (ab + ac + bc)y + abc$

---

16. a) $\dfrac{10\times11\times21}{6}$

b) This should read $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2$

$$\frac{n\times(n+1)\times(2n+1)}{6} \rightarrow \frac{n(n+1)(2n+1)}{6}$$
